ASSET MARKET LINKAGES IN CRISIS PERIODS

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Abstract—We characterize asset return linkages during periods of stress by an extremal dependence measure. Contrary to correlation analysis, this nonparametric measure is not predisposed toward the normal distribution and can allow for nonlinear relationships. Our estimates for the G-5 countries suggest that simultaneous crashes between stock markets are much more likely than between bond markets. However, for the assessment of financial system stability the widely disregarded cross-asset perspective is particularly important. For example, our data show that stock-bond contagion is approximately as frequent as flight to quality from stocks into bonds. Extreme cross-border linkages are surprisingly similar to national linkages, illustrating a potential downside to international financial integration.

I. Introduction

DO different financial markets crash jointly, or is a fall of one a gain for another? The answer to this question is crucial for our view on the stability of international financial markets and any systemic risk related to these markets. The more markets crash simultaneously, the more in danger are even large banks that hold widely diversified trading portfolios, possibly also threatening the payment and settlement process. The number of markets affected by a crisis situation may also determine the severity of any real effects that might follow. Recent financial crises in emerging market economies have again drawn attention to these issues. Market participants, policymakers, and academics frequently point to the perceived occurrence of contagion, witness terminology like the "Asian flu." Others highlight joint shocks and macroeconomic fluctuations, triggering simultaneous crises in several markets or countries.

The phenomenon of financial-market crises spilling over to other countries was first systematically studied by Morgenstern (1959, ch. X).[1] He examined the effects of 23 stock market panics on foreign markets and explicitly referred to the "statistical extremes" of the stock market movements. The more recent econometric literature uses correlation analysis, often based on ARCH-type models. This literature asks whether stock-market comovements become stronger during crashes than in noncrash times. It also investigates the direction of international spillovers. Representative articles of this literature are King and Wadwhani (1990), Hamao, Masulis, and Ng (1990), Malliaris and Urrutia (1992), Lin, Engle, and Ito (1994), and Susmel and Engle (1994). There is also some empirical work on whether currency crises are contagious, notably in Eichengreen, Rose, and Wyplosz (1996), Sachs, Tornell, and Velasco (1996), and Kaminsky and Reinhart (2000). However, there is very little work on bond market spillovers.[2]

The present paper adds a new perspective to the linkages between asset markets, by studying comovements between different types of assets and by using a novel methodology. In contrast to the existing literature, we do not only study the connection between, say, different stock markets during times of stress; we explicitly focus on the linkages between stock and government bond markets. Thus, apart from studying phenomena like contagion or joint crashes of stocks, we look into phenomena such as *flight to quality,* by which we mean a crash in stock markets accompanied by a boom in government bond markets. Extreme cross-asset linkages are important for the analysis of international financial stability, for they have a bearing on the overall, or *systemic,* reach that contagion or joint crashes can have. We are not aware of any other hard quantitative examination of cross-asset crisis linkages, including the flight-to-quality phenomenon.

The methodological novelty is that we do not use conditional correlation analysis. We directly measure and report the expected number of market crashes conditional on the event that at least one market crashes. Studies which rely on conditional correlation analysis usually do report the amount of correlation, but stop short of reporting the information that has more direct economic meaning. In our opinion, the correlation measure is only an intermediate step in obtaining a measure such as the likelihood of a crash spillover. The conditional correlation, moreover, is strongly predisposed toward the multivariate normal distribution. As our empirical analysis below shows, however, the multivariate normal dramatically underestimates the frequency of extreme market spillovers. Boyer, Gibson, and Loretan

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[1] We thank Charles Goodhart for pointing us to this historical reference.

[2] We know of only two studies that systematically address international bond-market spillovers in volatile periods, namely Borio and McCauley (1996) and Domanski and Kremer (2000). In contrast with recent advances in the theoretical analysis of (for example) bank contagion, there are surprisingly few theoretical attempts to explicitly model crisis linkages between different securities markets. The published literature comprises King and Wadhwani (1990), Calvo and Mendoza (2000), Kodres and Pritsker (2002), and Kyle and Xiong (2001). For a comprehensive survey of the contagion literature, see de Bandt and Hartmann (2000).

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(1997) demonstrate for the bivariate normal that the conditional correlation measure varies considerably with the conditioning sets.[3] In addition, this variation can easily be shown to be different for different classes of multivariate distributions; worse, the conditional crash correlation can be 0 even if there is a high spillover probability. For all these reasons, we do not regard the statistical concept of correlation as an unambiguous measure of the economic interdependence between markets during times of stress.

This paper instead characterizes the return linkages between asset markets in periods of crisis by a measure from statistical extreme-value analysis that captures the dependence structure of multivariate distributions far away from the center. It turns out that this limiting dependence measure can be described by a single function that exists under fairly general conditions. In contrast to correlation-based approaches toward measuring market linkages, the probability law of the joint return process can be left unspecified, because we use a nonparametric estimator for the limiting dependence function. From these nonparametric estimates of the limiting dependence function we derive estimates for the expected number of market crashes (or the probability of a simultaneous crash) given that at least one market crashes. Thus market linkages in crisis periods are measured directly in the economically relevant money metric and associated probabilities, and we do not make the detour via correlations.

The methodology is then used to analyze the linkages within and between equity and bond markets in the G-5 industrial countries in times of market turmoil.[4] Our results indicate small but nonnegligible cross-asset market linkages in times of stress. The strongest extreme linkages are between different national equity markets, and the flight-to-quality phenomenon is approximately as frequent as simultaneous crashes of stock and bond markets. Whereas single bond or stock market crashes are relatively rare events happening once or twice a human lifetime, the conditional probabilities of having a crash (or boom) in a market given that one occurred in another market are quite high. Interestingly, cross-border linkages are not weaker than domestic linkages. Whereas these results confirm that in the era of free capital flows and globalization, surveillance of financial market stability cannot stop at national borders, they also suggest that there are some limits to how widely contagion can spread.

[3] Forbes and Rigobon (2002) show a similar conditional correlation bias in the context of the linear regression model and propose a correction for it.

[4] Related analyses for a single type of contract have recently been carried out on foreign exchange data by Straetmans (1998) and Starica (1999), and on major stock markets by Straetmans (2000), Longin and Solnik (2001) and Poon, Rockinger, and Tawn (2001). For analyses of stock market comovements using regime-switching volatility models, see Ramchand and Susmel (1998). For studies of regular noncrisis cross-asset interdependence, see, for example, Longin and Solnik (1995), Fleming, Kirby, and Ostdiek (1998), or Bodart and Reding (1999).

The paper is organized as follows. The next section introduces our nonparametric asymptotic tail dependence measure for extreme financial market comovements. Section III presents the way in which this extreme linkage measure can be estimated in two steps, a univariate one and a bivariate one. Some related testing techniques are discussed in section IV. Section V contains the results from applying this approach to weekly G-5 country stock and bond market returns. The empirical analysis follows the two steps described in section III, first comparing extreme returns in stock and bond markets and then detailing national cross-asset linkages, cross-border linkages within the same asset class, and finally cross-border cross-asset linkages. Conclusions are drawn in section VI. Four appendices provide details about the derivation of a test statistic for extreme dependence, the small-sample properties of the tests used in the main body of the paper, the small-sample properties of our extreme linkage measure, and the data employed.

II. Extreme Linkages: Probability Theory

Suppose one is interested in measuring the expected number of market crashes given that at least one market crashes (or booms, as in the flight-to-quality case). This measure reflects how many markets are on average drawn down when one market crashes. Consider the case of two markets with random returns X and Y. Let x and y be the quantiles (or *thresholds*) above which we speak of a market boom or crash (in case of a loss). To study market crashes we adopt the convention of taking the negative of a return, so that we can study all extreme events in the first quadrant. Let κ stand for the number of markets with extreme returns. Our extreme linkage indicator is the conditional expectation $E[\kappa|\kappa \geq 1]$. From elementary probability theory (starting from the standard definition of conditional probability) we have that

$$\frac{P\{X > x, Y \leq y\} + P\{X \leq x, Y > y\} + 2P\{X > x, Y > y\}}{P\{X > x \text{ or } Y > y\}} = E[\kappa|\kappa \geq 1] = \frac{P\{X > x\} + P\{Y > y\}}{P\{X > x \text{ or } Y > y\}}, \tag{1}$$

with $P\{X > x \text{ or } Y > y\} = 1 - P\{X \leq x, Y \leq y\}$. Notice also that $E[\kappa|\kappa \geq 1] = P\{\kappa = 2|\kappa \geq 1\} + 1$, so that an alternative interpretation of our extreme linkage indicator is in terms of (1 plus) the conditional probability that both markets crash given that at least one market crashes. For higher dimensions than two $E[\kappa|\kappa \geq 1]$ is still equal to the ratio of the sum of the marginal excess probabilities divided by the joint failure probability. The measure $P\{\kappa = 2|\kappa \geq 1\}$ is however not as easily extended to higher dimensions.

The question is how $E[\kappa|\kappa \geq 1]$ can be calculated in practice. Within the framework of the multivariate normal distribution this would be a trivial exercise, because only the first two moments have to be estimated. In the introduction we argued, however, that the framework of the multivariate normal and the associated correlation structure may not be suitable for analyzing extreme linkages between asset markets. To be able to break away from specific distributional assumptions, we investigate $E[\kappa|\kappa \geq 1]$ when the conditioning quantiles x and y become very large. To this end, define the upper quantile functions for the returns X and Y respectively as

$$Q_1(tu) = (1 - F_1)^{-1}(tu),$$
$$Q_2(tv) = (1 - F_2)^{-1}(tv), \tag{2}$$

for some small but positive values u, v and a scaling parameter t. Choose u, v, and t such that tu and tv are smaller than 1 and thus interpretable as excess probabilities. Moreover, set $Q_1(tu) = x$ and $Q_2(tv) = y$, which are the original crash levels from equation (1) that we are interested in. In other words, we have inverted the cdf so as to work out the asymptotic equivalent of our linkage measure in terms of the (small) probabilities of having very extreme returns. This will prove convenient below for bringing out some nice properties of the measure.

Upon substituting the above quantile functions into equation (1), one obtains the following asymptotic equivalent for that equation:

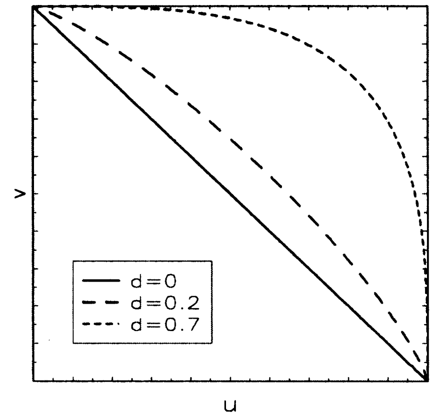
$$\lim_{t\to+0} E[\kappa|\kappa \geq 1] = \lim_{t\to+0} \frac{t^{-1}P\{X > Q_1(tu)\} + t^{-1}P\{Y > Q_2(tv)\}}{t^{-1}[1 - P\{X \leq Q_1(tu),\ Y \leq Q_2(tv)\}]} = \frac{u+v}{l(u,\ v)}. \tag{3}$$

The result of letting t converge to 0 is that the excess probabilities tu, tv also tend to 0, and hence the quantiles become very large. Thus equation (3) says that very far from the origin our linkage indicator is asymptotically equal to the sum of the marginal probabilities divided by a limit function $l(u, v)$. The function $l(u, v)$ in the denominator in equation (3) is the so-called *stable tail dependence function* (STDF) and was introduced by Huang (1992). Multivariate extreme value theory deals with existence conditions, properties, and estimators for this function; see Huang (1992) or de Haan and de Ronde (1998).[5] The curvature of $l(u, v)$ completely determines the dependence structure between X and Y in the tail area. Basic properties of $l(u, v)$ are its linear homogeneity and the inequality

$$\max(u, v) \leq l(u, v) \leq u + v. \tag{4}$$

[5] The limit function exists if the joint extremal returns from X and Y lie in the domain of attraction of a multivariate extreme-value distribution.

FIGURE 1.—UNIT CONTOUR LINES OF THE BIVARIATE GUMBEL DISTRIBUTION FUNCTION



Note: The variables on the axes are interpretable as asymptotic excess probabilities; see equation (6). The linear homogeneity of $l(u, v)$ implies that a single unit contour line $l(u, v) = 1$ visually characterizes the tail dependence structure for each value of the dependence parameter d. The solid line corresponds with full asymptotic independence, whereas the dashed lines reflect increasing dependence in the tails.

Equality holds on the left-hand side if X and Y are completely dependent in the tail area, whereas it holds on the right-hand side if X and Y are independent in the tail area. Note that independence means that for all Q_1 and Q_2

$$P\{X < Q_1,\ Y < Q_2\} = P\{X < Q_1\}P\{Y < Q_2\}, \tag{5}$$

whereas tail independence only requires this factorization to hold asymptotically. Thus it may well be that nonextreme return pairs are dependent although their extremes are asymptotically independent. The bivariate normal distribution with $\rho \in (-1, 1)$ and $\rho \neq 0$ constitutes such a case, for example.

The linear homogeneity of $l(u, v)$ implies that all contour lines exhibit the same shape toward the origin. Thus it suffices to plot the *unit* contour line in order to get a full graphical representation of dependence in the bivariate tail. Moreover, the combined properties of $l(u, v)$ imply that the unit contour line is concave toward the origin, ending at (1, 0) and (0, 1) and being fully contained in the uniform rectangular. For illustrative purposes consider the bivariate cdf

$$F(x, y) = \exp[-(x^{-1/(1-d)} + y^{-1/(1-d)})^{1-d}],$$
$$0 \leq d < 1, \quad x, y \in [0, \infty),$$

with marginal Frechet distributions (see, for example, Gumbel, 1958). Upon taking the limit in the denominator of equation (3), the STDF of this distribution boils down to

$$l(u, v) = (u^{1/(1-d)} + v^{1/(1-d)})^{1-d}.$$

The dependence parameter d determines the degree of tail dependence between X and Y. Figure 1 plots unit contour lines (the asymptotic marginal probabilities on the axes are normalized to u, $v \in [0, 1]$) for this pair of random variables for different values of d. The lower bound of asymptotic independence is hit when $d = 0$, and the unit contour line coincides with the uniform rectangular (complete asymptotic dependence) when d approaches 1.

The STDF relates marginal and joint probabilities as follows. First define the excess probabilities $p_1 = P\{X > x\}$, $p_2 = P\{Y > y\}$, and $p_{12} = 1 - P\{X \le x, Y \le y\}$, for ease of reference. Exploiting the homogeneity property, one can easily show that the bivariate excess probability p_{12} and the marginal probabilities p_1 and p_2 are related via the STDF. For sufficiently small $t > 0$,

$$l(u, v) \approx t^{-1}[1 - P\{X \le Q_1(tu), Y \le Q_2(tv)\}]. \quad (6)$$

Choose $tu = p_1$ and $tv = p_2$, so that $l(u, v) = l(t^{-1}p_1, t^{-1}p_2)$. Use the linear homogeneity of the STDF to write $tl(t^{-1}p_1, t^{-1}p_2) = l(p_1, p_2)$. Hence, for small values of p_1 and p_2, approximately,

$$l(p_1, p_2) \approx p_{12}. \quad (7)$$

Thus the joint probability p_{12} only depends on the marginal probabilities p_1 and p_2 and the dependence function $l(\cdot, \cdot)$. The linkage measure can thus be expressed as

$$E[\kappa|\kappa \ge 1] = \frac{p_1 + p_2}{p_{12}} \approx \frac{p_1 + p_2}{l(p_1, p_2)}. \quad (8)$$

Assume for example that $p_1 = p_2 = p$. Then, approximately,

$$E[\kappa|\kappa \ge 1] \approx \frac{2p}{l(p, p)} = \frac{2}{l(1, 1)}. \quad (9)$$

If both returns are completely dependent in the tails, that is, $l(1, 1) = \max(1, 1)$, then $E[\kappa|\kappa \ge 1] \approx 2$ and the markets cocrash with certainty. But without extreme comovements in the two markets, $E[\kappa|\kappa \ge 1] \approx 1$, because $l(1, 1) = 2$.

III. Extreme Linkages: Estimation

The conditional expectation (1) is estimated by a two-step estimation procedure. In the first step one estimates the marginal extreme quantile cum probability combinations (p_1, p_2). In the second step one imputes these univariate probability estimates into an estimator for the tail dependence function $l(\cdot, \cdot)$ in order to obtain an estimator for p_{12}. The estimation procedure therefore essentially exploits equation (7).

Univariate excess probability estimation uses the stylized fact that asset return distributions exhibit heavy tails. Loosely speaking, this implies that the excess probability as a function of the corresponding quantile can be approximately described by a power law. The defining characteristic of these distributions is the property of regular variation at infinity,

$$\lim_{q\to\infty} \frac{1 - F(qx)}{1 - F(q)} = x^{-\alpha}, \qquad x > 0, \quad \alpha > 0. \quad (10)$$

From this property it directly follows that such distributions (for example, the Student t) have bounded moments only up to α, where α is known as the tail index. In contrast, distributions with exponentially decaying tails or with finite endpoints have all moments bounded.

Univariate excess probabilities for fat-tailed marginals can be estimated by using the semiparametric probability estimator from de Haan et al. (1994):

$$\hat{p}_q = \frac{m}{n}\left(\frac{X_{n-m,n}}{q}\right)^{\alpha}, \quad (11)$$

where the *tail cutoff point* $X_{n-m,n}$ is the $(n - m)^{\text{th}}$ ascending order statistic (or, loosely speaking, the m^{th} smallest return) from a sample of size n such that $\lim[1/m(n)] = 0$ but $m = o(n)$, and where the extreme (probability quantile) combination $(\hat{p}_q, q)$ is such that $q > X_{n-m,n}$.[6] An important aspect of the estimator $\hat{p}_q$ is that it can extend the empirical distribution function outside the domain of the sample by means of its asymptotic Pareto tail from equation (10). The estimator (11) is conditional upon the tail index α. We estimate the tail index by means of the popular Hill (1975) estimator:

$$\hat{\gamma} = \frac{1}{m}\sum_{j=0}^{m-1} \ln\left(\frac{X_{n-j,n}}{X_{n-m,n}}\right), \quad (12)$$

where m has the same value and interpretation as in equation (11) and $\hat{\alpha} = 1/\hat{\gamma}$ stands for the estimated tail index. Further details are provided in Jansen and de Vries (1991) and the recent monograph by Embrechts, Klüppelberg, and Mikosch (1997).

The estimation of the *bivariate* excess probability p_{12} either requires adopting a specific functional form for the STDF, as in Longin and Solnik (2001), or proceeding semiparametrically. Because there does not exist a unique parametrization for the STDF, we pursue a *semiparametric* estimation method based on the highest order statistics. Let $t = k/n$ in equation (6), such that $\lim[1/k(n)] = 0$, but $k = o(n)$. (The role of the nuisance parameter k corresponds to that of m in the univariate estimation step.) Because the marginal probability estimates are available from the univariate step, we can also replace (u, v) by $(\hat{p}_1, \hat{p}_2)$:

$$l(\hat{p}_1, \hat{p}_2) = \lim_{n\to\infty} \frac{n}{k} P\left\{X \ge Q_1\left(\frac{k\hat{p}_1}{n}\right) \text{ or } Y \ge Q_2\left(\frac{k\hat{p}_2}{n}\right)\right\}. \quad (13)$$

[6] In financial risk management the scaling parameter q is usually referred to as the "value at risk (VaR)," and it is often used in a reversed fashion: what is the VaR $\hat{q}_p$ for a given probability p?

In order to turn this expression into an estimator for $l(\cdot, \cdot)$, we replace $P\{\cdot\}$, $Q_1(\cdot)$, and $Q_2(\cdot)$ by their empirical counterparts, so that approximately

$$\hat{l}(\hat{p}_1, \hat{p}_2) \approx \frac{n}{k}\frac{1}{n}\sum_{i=1}^{n} \mathbf{1}\{X_i > X_{n-[k\hat{p}_1],n} \text{ or } Y_i > Y_{n-[k\hat{p}_2],n}\}, \tag{14}$$

where $\mathbf{1}$ denotes the indicator function and where $[x]$ is the integer satisfying $x \leq [x] < x + 1$. So, loosely speaking, the estimator of l boils down to counting the instants at which one or both of the markets experience an extreme return within a given sample period.

Because the marginal probability arguments of the STDF are typically smaller than the reciprocal of the sample size n, the empirical probability measure (14) is not operational. However, one can increase the number of excesses by scaling up the marginal probabilities in equation (14) by a factor $\lambda > 1$ and exploiting the linear homogeneity property $\hat{l}(\hat{p}_1, \hat{p}_2) \approx \lambda^{-1}\hat{l}(\lambda\hat{p}_1, \lambda\hat{p}_2)$.[7] Huang (1992) proposed to transform the marginal probability pair $(\hat{p}_1, \hat{p}_2)$ to polar coordinates $(\hat{r}\cos\hat{\phi}, \hat{r}\sin\hat{\phi})$ such that $\lambda = 1/\hat{r}$.[8] The polar coordinate representation of the empirical measure (14) is

$$\begin{aligned}\hat{p}_{12} &\approx \hat{r}\hat{l}(\cos\hat{\phi}, \sin\hat{\phi}) \\ &= \frac{1}{k}\hat{r}\sum_{i=1}^{n} \mathbf{1}(X_i > X_{n-[k\cos\hat{\phi}],n} \text{ or } Y_i > Y_{n-[k\sin\hat{\phi}],n}).\end{aligned} \tag{15}$$

This estimator evaluates l on the unit circle, which is convenient in that it is based on a larger set of observations than equation (14). Notice that this procedure of counting coexceedances is easily applicable in higher dimensions. An estimator for the expected number of simultaneous crashes $E[\kappa|\kappa \geq 1]$ directly follows if one replaces p_1, p_2, and p_{12} in equation (8) by their respective estimators in equations (11), (12), and (15):

$$\hat{E}[\kappa|\kappa \geq 1] = \frac{\cos\hat{\phi} + \sin\hat{\phi}}{\hat{l}(\cos\hat{\phi}, \sin\hat{\phi})}. \tag{16}$$

The entire estimation procedure thus depends on three estimators (11), (12), and (15) that are easy to calculate. Conditional on the proper choice of the nuisance parameters m and k, the three estimators are asymptotically normally distributed. Goldie and Smith (1987) and Huang (1992) propose to pick m and k in a range that minimizes the respective asymptotic mean squared errors (MSEs).[9] Consequently, minimizing the sample MSE is the appropriate selection criterion. In small samples best practice is to plot the estimators as a function of the threshold, that is, $\hat{\gamma} = \hat{\gamma}(m)$ and $\hat{l} = \hat{l}(k)$, and to select m and k in the region over which the estimators tend to be constant.[10]

IV. Extreme Linkages: Hypothesis Testing

The asymptotic normality of the estimators enables some straightforward hypothesis testing. Hall (1982) showed for $m/n \to 0$ as $m, n \to \infty$ that the statistic $\sqrt{m}[\hat{\gamma}(m)/\gamma - 1]$ is asymptotically standard normally distributed. A test for the equality of tail indices can thus be based on the following T-statistic:

$$T = \frac{\hat{\gamma}_1(m_1) - \hat{\gamma}_2(m_2)}{\sigma[\hat{\gamma}_1(m_1) - \hat{\gamma}_2(m_2)]}, \tag{17}$$

which converges to a standard normal distribution in large samples. The denominator's standard deviation is calculated as the standard deviation of the bootstrapped difference $\hat{\gamma}_1 - \hat{\gamma}_2$ (we chose the number of bootstraps equal to 600). Small-sample properties of tail index estimators are studied in (for example) Dekkers and de Haan (1989).

Huang (1992) has proven the asymptotic normality of the estimator (15) for $k/n \to 0$ as $k, n \to \infty$. We use this to compare the amount of extreme dependence across different return quadrants. For a pair of stock and bond markets, we can test whether a cocrash (for example, through contagion) is more likely than flight to quality from stocks into bonds or vice versa by calculating the following Z-statistic:

$$Z = \frac{\hat{l}_{\mathrm{CO}}(k_1) - \hat{l}_{\mathrm{FTQ}}(k_2)}{\sigma[\hat{l}_{\mathrm{CO}}(k_1) - \hat{l}_{\mathrm{FTQ}}(k_2)]}, \tag{18}$$

which has a standard normal distribution in large samples. The denominator's standard deviation is calculated as the standard deviation of the bootstrapped difference $\hat{l}_{\mathrm{CO}} - \hat{l}_{\mathrm{FTQ}}$ (again with the number of bootstraps set equal to 600).[11] In equation (18) the subscripts CO and FTQ on the STDF estimates refer to stock cum bond market cocrashes and stock market crashes cum bond market booms (flight to quality), respectively.

It is also of interest to *pretest* for the presence of asymptotic dependence in specific market pairs. Peng (1999)

[7] Appendix C provides a discussion of the approximate character of linear homogeneity in finite samples. More specifically, it is shown that the finite-sample character of the estimator $\hat{l}$ together with the inward shift of the quantiles on assuming homogeneity creates an upward bias in the extreme linkage estimates for E. This bias may be empirically relevant in small samples.

[8] Consistent estimators for the angle and radius are provided by $\hat{\phi} = \arctan(\hat{p}_2/\hat{p}_1)$ and $\hat{r} = \sqrt{\hat{p}_1^2 + \hat{p}_2^2}$, respectively.

[9] If the value chosen for m or k is too small, then the respective estimate is inefficient, because not enough observations are used. If it is too large, then one incurs a bias, because the interior of the distribution is measured rather than the tails. For a given sample there exists an intermediate range of values for which bias squared and variance vanish at the same rate, but outside this range one of the two parts dominates and the asymptotic MSE is higher.

[10] Plots are available from the authors upon request. See, for example, Embrechts et al. (1997) and de Haan and de Ronde (1998) for this widespread practice in small samples.

[11] The tail dependence function is evaluated at (1, 1) along the 45-degree line for convenience.

proposes a testing procedure that starts from a general second-order expansion for $l(u, v)$ in finite samples:

$$l(u, v) \approx u + v - c(u, v)t^{1/\eta - 1}[1 + O(t^{\beta})],\quad \eta \in (0, 1], \quad \beta > 0. \tag{19}$$

The value of the tail dependence coefficient η governs whether extremal returns are asymptotically dependent or not. The function $c(u, v)$ is a concavity term that is retained far into the bivariate tail ($t \to 0$) only if the exponent $\eta = 1$, indicating asymptotic dependence. If the joint distribution is asymptotically independent, then $\eta < 1$. For example, in case of independence, $\eta = \frac{1}{2}$ and $c(u, v) = uv$, so that $l(u, v) \approx u + v - uvt$ [and hence as $t \to 0$, $l(u, v) = u + v$ asymptotically]. A heuristic derivation of an estimator $\hat{\eta}$ for the tail dependence coefficient is provided in appendix A. This estimator enables one to test the $H_0 : \eta = 1$ (asymptotic dependence) against the $H_1 : \eta < 1$ (asymptotic independence) by means of the statistic

$$W = \frac{\hat{\eta} - 1}{\sigma(\hat{\eta})}, \tag{20}$$

which is asymptotically normally distributed under H_0. The asymptotic standard error $\sigma(\hat{\eta})$ can be expressed in terms of the limiting dependence function and its partial derivatives (see Peng, 1999).

Because only a fraction of our original data set enters the estimators and test statistics for extreme dependence patterns, we investigated the small-sample properties of the tools developed. In appendix B we describe Monte Carlo simulations to discuss the small-sample behavior of both the Z-statistic and the W-statistic. The results show that the left-tail critical values for Z lie reasonably close to their limiting values, whereas we find some evidence for size distortions in W. However, if one uses the small-sample critical values instead of their asymptotically normal counterparts, the empirical section's test results for asymptotic dependence hardly change. As for the small sample power of W, it might be low if η is close to 1 under the alternative hypothesis of asymptotic independence.[12] Heffernan (2001), however, shows that a majority of parametric models with asymptotically independent tails exhibit a tail dependence parameter η equal to $\frac{1}{2}$, making the local alternatives problem relatively unlikely to occur.

Regardless whether the data are asymptotically independent or not, in the end we are interested in the accuracy of the linkage measure $E[\kappa|\kappa \geq 1]$ evaluated for large but finite crash levels. Appendix C contains both analytical results and Monte Carlo evidence on the small-sample bias in the linkage estimator. We report simulation results on this bias for three data-generating processes: independent normal data, bivariate normal data that are asymptotically independent but exhibit nonzero correlation, and bivariate Pareto data that exhibit asymptotic dependence. It transpires that for the sample sizes we are working with and at the typical correlation level in our data, the estimates of the linkage measure provide a conservative upper bound for extreme market spillovers. Moreover, the bias seems more severe if the data are asymptotically independent. The simulations also show that with a data set 10 times larger than the current one, the bias disappears almost completely.

To conclude, from both the power and the size study of the W-test and from the performance of the linkage measure in small samples we infer that the results for the true data reported below provide an upper bound on the amount of extreme linkage between different financial markets. As we will argue below, the upper bound interpretation will make some of our results even stronger.

V. Extreme Linkages: Results for G-5 Countries

In this section we evaluate the extent of extreme comovements within and between stock and bond markets. The data consist of 663 (nonoverlapping) weekly stock and government bond returns for the G-5 market indices over the period 1987 to 1999. A detailed description of the data is given in appendix D. We start with the univariate stock and bond market extremes, eyeballing first indications for their joint occurrence. Then we turn to the systematic application of our extreme linkage measure and the tests of extreme dependence patterns.

A. *Extreme Returns in Stock and Bond Markets*

In the univariate step, we report tail index and quantile-probability estimates on the basis of equations (12) and (11). Table 1 contains information on the magnitude and timing of the most extreme in-sample events for stocks (panel A) and bonds (panel B). The table also gives the estimates of the tail index α and the accompanying tail probabilities conditioned on different quantile levels. Within both panels we further distinguish between the upper and the lower tails of the univariate return distributions in order to take into account possible asymmetries.[13]

From the table we see that extreme losses are generally much higher for stock indices than for government bond indices. Moreover the historical extremes point toward asymmetries in stock index returns: the (absolute) extreme loss returns consistently exceed the maximum positive returns.[14] When comparing the entries for stock and bond

[12] For example, take the case of the bivariate normal with correlation coefficient ρ. In this case $\eta = (1 + \rho)/2$, $\rho \in (-1, 1)$.

[13] For reasons of space the nuisance parameters m on which the Hill estimator and the tail probability estimates are conditioned are omitted from the table. The values, which varied between 10 and 50, are available from Table 1 of the underlying working paper (Hartmann, Straetmans, & de Vries, 2001).

[14] On Friday, October 16, 1987 the UK stock market remained closed due to a hurricane. Therefore, the weekly return in the UK for the period around Black Monday is calculated from Thursday to Friday, not from Friday to Friday. As a result the figure of a 25% fall (much higher than for the four other countries) might somewhat overstate the London crash in relative terms.

TABLE 1.—MINIMA, MAXIMA, TAIL INDEX, AND UNIVARIATE TAIL PROBABILITY ESTIMATES FOR WEEKLY G-5 ASSET RETURNS (1987–1999)

A. Stocks

	Left Tail				Right Tail			
			$P\{X < -q\}$				$P\{X > q\}$	
Country	Min (%)	$\hat{\alpha}$	−15%	−20%	max (%)	$\hat{\alpha}$	15%	20%
GE	−13.81 (10/2/98)	3.32	0.05013	0.01926	11.57 (10/16/98)	4.68	0.00591	0.00153
FR	−11.12 (10/23/87)	3.24	0.03320	0.01308	9.61 (10/16/98)	5.68	0.00202	0.00039
UK	−24.83 (10/23/87)	3.76	0.01129	0.00382	8.19 (9/18/92)	4.54	0.00340	0.00090
US	−13.60 (10/23/87)	2.33	0.05009	0.02559	7.25 (10/16/98)	5.56	0.00086	0.00017
JP	−13.32 (10/23/87)	3.07	0.05337	0.02209	11.18 (8/21/92)	4.61	0.01495	0.00397

B. Bonds

	Left Tail				Right Tail			
			$P\{X < -q\}$				$P\{X > q\}$	
Country	Min (%)	$\hat{\alpha}$	−6%	−8%	max (%)	$\hat{\alpha}$	6%	8%
GE	−6.61 (12/30/94)	5.39	0.06564	0.01393	3.77 (1/9/98)	5.57	0.00749	0.00151
FR	−5.14 (10/28/94)	6.01	0.03202	0.00568	5.21 (10/23/87)	5.58	0.00568	0.00114
UK	−6.35 (10/9/98)	3.91	0.03412	0.01109	4.81 (4/10/92)	4.46	0.02771	0.00767
US	−5.41 (5/15/87)	3.45	0.08996	0.03301	9.77 (10/23/87)	3.72	0.04323	0.01480
JP	−5.08 (12/25/98)	4.58	0.03497	0.00936	4.30 (11/6/87)	2.84	0.05164	0.02277

Note: $\hat{\alpha}$ is the reciprocal of the Hill estimator in equation (12). Marginal probabilities P are conditioned on extreme stock and bond market return quantiles near the historical sample boundaries and are calculated using equation (11). Weekly prices were sampled on Fridays.

markets, the timing of the extreme events, as recorded in parentheses, suggests the presence of cocrashes and flight-to-quality effects during periods of market turbulence. Indeed, all stock markets covered, except for Germany's, reached historically low returns in the week of Black Monday. As an aside, the table also shows that three stock markets (FR, GE, US) exhibited parallel record gains as a consequence of a major rebound in mid October 1998 following the Russian and LTCM crises, and two stock markets (JP, UK) exhibited comparable record gains around the September 1992 European currency crisis.[15] The casual extreme linkage evidence is less clear for the bond markets. For example, none of the largest bond index corrections occurred during the February-to-June 1994 fixed-income market turmoil, and only the United Kingdom experienced a record slump in the aftermath of the LTCM crisis. The October 1987 rallies in the French and U.S. bond markets (and perhaps also the Japanese rally two weeks later) are suggestive of a flight-to-quality effect from stocks into government bonds.

We turn to the remaining columns in table 1. The left-tail index estimates are highest for the bond returns, indicating thinner lower tails than for the stocks. This reflects the more limited downside risk of government-bond investments. Moreover, and in contrast to the bond series, the point estimates $\hat{\alpha}$ for the left tail of the stock index series are lower than their right-tail counterparts. This is consistent with the observed asymmetry between the minimum and maximum stock returns reported in the left part of the table and also squares well with the results reported by Longin and Solnik (2001). Using the T-test as defined in equation (17), we formally tested for equality of tail indices across lower and upper tails and across assets. It turned out that only in the French and U.S. stock markets are the larger sizes of left tails statistically significant in our sample. And again, only for France is the left stock market tail significantly thicker than the left bond market tail. In almost all other cases the null hypothesis of equal tail indices could not be rejected.[16]

The economic issue of interest, both for the general assessment of financial market stability and for financial institutions' risk management, is the likelihood of the

[15] The October 1998 stock market rallies occurred in an environment of high uncertainty and volatility in international financial markets. They seem to be directly related to the U.S. Fed's emergency interest rate cut on October 15. On that day the Federal Open Market Committee decided to reduce both the Fed funds target rate and the discount rate by 25 basis points (see Federal Reserve Board, 1998a). This action surprised markets, because it happened by means of an FOMC telephone conference between the scheduled meetings of September 29 and November 17 (see Federal Reserve Board, 1998b, conference call). Three weeks before, the Federal Reserve Bank of New York had coordinated a private-sector bailout of the Long-Term Capital Management hedge fund (see Greenspan, 1998). During the same period exchange rates also experienced historical jumps, notably in the form of an unprecedented yen appreciation (see Hartmann, Straetmans, & de Vries, 2003, table 1).

[16] Further details on test results are reported in the underlying working paper (Hartmann et al., 2001, appendix 1) and are available on request.

TABLE 2.—CROSS-BORDER EXTREME LINKAGES WITHIN STOCK AND BOND MARKETS (1987–1999)

	Stocks				Bonds			
			E				E	
Pair	W	ρ	−15%	−20%	W	ρ	−6%	−8%
GE-FR	0.601	0.686	1.283	1.263	−0.251	0.600	1.194	1.164
GE-UK	−0.457	0.575	1.149	1.130	−0.859	0.438	1.124	1.109
GE-US	−1.206	0.470	1.140	1.148	−0.466	0.291	1.075	1.090
GE-JP	−0.437	0.314	1.205	1.216	−1.307	0.198	1.041	1.051
FR-UK	−1.131	0.589	1.220	1.208	−0.758	0.491	1.071	1.085
FR-US	−1.156	0.497	1.175	1.201	−1.059	0.363	1.077	1.049
FR-JP	−1.159	0.322	1.148	1.142	−0.691	0.129	1.030	1.023
UK-US	−1.404	0.546	1.135	1.079	−0.512	0.425	1.075	1.100
UK-JP	−1.370	0.361	1.071	1.057	−1.340	0.184	1.094	1.104
US-JP	−0.888	0.328	1.104	1.119	−0.876	0.164	1.079	1.079

Note: W refers to Peng's asymptotic dependence test in equation (20). The extreme linkage measure E is conditioned on extreme quantiles near the historical sample boundaries and is estimated using equation (16). Unconditioned (full-sample) correlation estimates are reported for comparison.

extreme returns as reflected by the tail probabilities. The reported probabilities are expressed over a yearly time horizon by multiplying the weekly probability estimates from equation (11) by a factor of 52. First, note that the excess probabilities are conditioned on different quantiles for stocks and bonds with an eye toward the historical minima and maxima displayed in table 1. One reason why conditioning crash levels are deliberately chosen to be in the vicinity of the historical extremes is to get a feeling for the probability of worst-case losses over a longer period of time such as a year in our case. Another reason, perhaps even more important for policymakers, is that there can be little doubt that such extremes constitute severe securities market crisis situations.

An interpretation of the loss levels in the table is as follows. For example, the entry 0.02559 for the left tail of the U.S. stock index implies that a 20% weekly crash on average happens once per 1/0.02559 ≈ 39 years (a 15% decline, a figure closer to the one for the week of the 1987 stock market crash, would happen approximately every 20 years). In U.S. government bond markets a weekly decline of 8% is expected to occur approximately once per 30 years (and a 6% decline—close to the historical negative extreme in table 1—once every 11 years). In other words, such crashes are rare events, but not so uncommon as one might believe. Compare these estimates with a normal-distribution-based estimate, which predicts weekly crashes in U.S. common stock of 20% or more to happen only once per 31.5×10^{20} years!

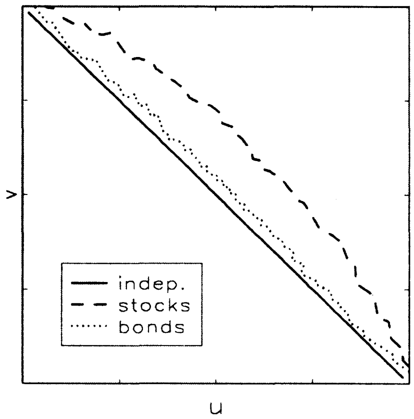
B. Extreme Comovements Within and Between Stock and Bond Markets

In the bivariate step we first look separately at extreme linkages between G-5 stock markets and between G-5 bond markets (table 2); then we consider domestic and international cross-asset market linkages (table 3). Both tables contain the W-pretest on asymptotic dependence [equation (20)], the correlation coefficient ρ, and the estimated linkage measure [equation (16)]. The linkage estimates are conditioned on the same extreme stock and bond return quantiles as in table 1 (see the discussion of their levels in the previous section). Displaying two crash levels for each asset class will allow for some sensitivity analysis. Also, the quantile pairs on which the linkage estimates are conditioned in tables 2 and 3 are chosen so that the corresponding univariate excess probabilities (table 1) are of similar order of magnitude and equation (9) approximately applies.

Table 2 reports estimation and testing results on extreme linkages within the same asset class (across borders). First, the W-test never rejects the null of asymptotic dependence at the 5% significance level for either of the two asset categories. Appendix B suggests some caution when testing asymptotic dependence in small samples, particularly regarding the use of asymptotic (normal) critical values. However, when using the small-sample critical values for the W-test in table 2 (and also in table 3 below), hardly any of the test results change. Second, the extreme linkage estimates are only marginally altered when shifting the conditioning quantiles further outward. Also, with the exception of the continental European country pair GE-FR, which exhibits the most highly interlinked stock and bond markets, no clear geographical patterns of crisis linkages can be discerned. Most strikingly, however, and regardless of the conditioning quantile pairs, extreme cross-border linkages at the lower tail are stronger within stock markets than within bond markets. For example, for the pair consisting of Germany (GE) and the United States (US), only 1 out of 6 (≈1/0.148) stock crashes is expected to be a 20% cocrash. However, only 1 out of 13 (≈1/0.075) bond crashes in the United States or Germany is expected to be a 6% cocrash.

To illustrate this in yet another way, we plot estimated contour lines for stock and bond tail dependence between Germany and Japan in figure 2 (the figure illustrates an empirical semiparametric application of the STDF introduced in section II, whereas figure 1 provided a theoretical illustration for a given parametric distribution). As a benchmark for comparison the linear contour line corresponding to asymptotic independence is also entered in the figure. The estimated contour lines reveal a degree of limiting depen-

FIGURE 2.—ESTIMATED UNIT CONTOUR LINES FOR STOCK AND BOND PAIRS (GERMANY, JAPAN)



Note: The estimator (14) is used to estimate unit contour lines (the dotted and dashed lines) by letting the angle ϕ vary over the first quadrant. The dashed and dotted lines represent the limiting dependence structures for the German-Japanese pair of stock markets and the German-Japanese pair of bond markets, respectively. The solid straight line corresponds with asymptotic independence and is included for comparison.

dence that is clearly higher for stock market pairs than for bond market pairs. The bond market contour lies close to the asymptotic independence benchmark case but is still significantly nonlinear (cf. the results in table 2 on significant asymptotic dependence between pairs of bond market returns). Note that the greater propensity of stock markets to extreme comovements than that of government bond markets is not an artifact created by the choice of conditioning quantiles, for the stock market quantiles are not less extreme compared to historical stock market experience than the bond market quantiles compared to bond market experience.

One may be tempted to interpret the potential for stock and bond cocrashes in Table 2 as small. However, compared to the unconditional univariate probability of experiencing a crash in a specific market (table 1), the (conditional) probability of having a crash in this market given that there is already one in another market is markedly higher. This illustrates the relevance of phenomena like contagion or joint crises as a consequence of a common shock. In other words, although severe securities market crises seem to be fairly rare events if one predicts them without using price information from other markets, it is not that unlikely for crashes to occur jointly once one market is hit by a crisis. Nevertheless, the spillover probabilities estimated do not appear very high in absolute terms either, rarely exceeding 20%. This means that whereas contagion or joint securities market crashes are of practical likelihood, they do not seem to be prevalent among the G-5 countries. This point may be further strengthened by noting the potential for an upward bias in the linkage estimator for small samples (see section IV and appendix C).

Finally, we also included full-sample correlations as a traditional linkage measure in table 2. One might easily interpret these as suggesting even higher linkages, but that is rather illusory. Suppose one applied the bivariate normal distribution to assess the extreme stock-stock and bond-bond market linkages, say, between France and Germany, the two most financially interlinked economies in table 2 (using the sample variances and correlations, with only the latter shown in the table). One would find that codependence is absent for stock markets in both countries at the 20% level. In fact, we only find codependence for crash magnitudes of 15% or lower (1 out of 228 stock crashes is then expected to be a cocrash). As for bonds, an 8% crash is expected to spread in only 1 out of 1,667 cases. Hence, the multivariate normal massively underrates extreme financial market linkages.[17]

Table 3 contains estimation and testing results on extreme linkages across asset classes, allowing for comparisons of stock-bond cocrash probabilities with the flight-to-quality phenomenon (the probability that a bond market booms, given that a stock market crashes). The table is further divided in two panels. The upper one (panel A) gives the results for stock-bond market linkages within a specific G-5 country, and the lower one (panel B) details international stock-bond market linkages between the G-5 countries. In the lower panel we adopt the convention that the first country mentioned has the stock market crash and the second country the bond market crash or boom. This allows us to treat the country pairs in panel B asymmetrically in terms of conditioning. This refinement enables us to look below at a new phenomenon, namely safe-haven behavior by investors.

The two left-side columns show results for the W-test of asymptotic dependence in the lower tails (W_{CO}) and between the lower and the upper tails (W_{FTQ}) using equation (20). The pretest statistics do not indicate a rejection of asymptotic dependence for most asset market pairs. Only for the GE-FR pair in row 6 is the hypothesis of asymptotic cocrashes between German stocks and French bonds rejected at the 5% level (but not for French stocks and German bonds, as shown in the row below). In other words, for most pairs of G-5 countries our data display statistically significant interdependence among financial markets during periods of crisis.

Given that there is asymptotic dependence both between the losses on stocks and bonds and between stock losses and gains in the bond markets, it is of some interest to test for the equality of the two effects using the Z-test from equation (18) in column 3.[18] The test shows that cocrashes dominate flight to quality in only 2 out of 25 cases (in both cases at the 1% significance level; see the two asterisks in the Z

[17] Due to the risk of biased correlation coefficients when conditioning on different ranges of the return distribution's support (Boyer et al., 1997; Forbes & Rigobon, 2002) and due to the extreme quantiles needed for calculating our conditional linkage indicator, we report the regular correlation coefficient for the whole distribution rather than conditional correlations in tables 2 and 3. All probabilities displayed in the text are derived from bivariate normals using these unconditional correlations.

[18] The Z-test value for the GE-FR pair is omitted from table 3, because its limit distribution is degenerate under asymptotic independence.

TABLE 3.—DOMESTIC AND INTERNATIONAL EXTREME STOCK-BOND LINKAGES: COCRASHES VERSUS FLIGHT TO QUALITY (1987–1999)

A. Domestic

	Test Statistics			Linkage Estimates				
					E_{CO}		E_{FTQ}	
Country	W_{CO}	W_{FTQ}	Z	ρ	(−20; −6)	(−20; −8)	(−20; 6)	(−20; 8)
GE	−0.881	−0.265	0.311	0.190	1.034	1.027	1.061	1.034
FR	−0.906	−0.974	−1.753	0.248	1.069	1.115	1.050	1.055
UK	−0.740	−0.463	−3.992*	0.217	1.040	1.059	1.057	1.073
US	−0.658	−0.906	−0.432	0.235	1.030	1.052	1.059	1.046
JP	−0.719	−1.376	0.099	0.051	1.061	1.092	1.036	1.050

B. Cross-Border

	Test Statistics			Linkage Estimates				
					E_{CO}		E_{FTQ}	
Pair	W_{CO}	W_{FTQ}	Z	ρ	(−20; −6)	(−20; −8)	(−20; 6)	(−20; 8)
GE-FR	−2.452*	−0.681	—	0.187	1.114	1.093	1.071	1.057
FR-GE	0.468	−0.534	−0.310	0.172	1.050	1.039	1.044	1.039
GE-UK	−1.271	−1.031	0.000	0.079	1.035	1.078	1.087	1.059
UK-GE	−0.362	−0.437	0.810	0.083	1.022	1.053	1.062	1.052
GE-US	−1.445	−0.314	−0.914	0.015	1.023	1.035	1.078	1.079
US-GE	−0.490	0	0.600	0.122	1.044	1.060	1.056	1.057
GE-JP	−0.450	−0.416	0.277	−0.056	1.045	1.096	1.090	1.068
JP-GE	−0.413	−0.861	0.683	−0.000	1.038	1.014	1.028	1.031
FR-UK	−1.107	0	−0.367	0.165	1.076	1.052	1.065	1.080
UK-FR	−1.013	−0.269	−0.874	0.102	1.049	1.068	1.068	1.051
FR-US	−0.473	−1.459	0.831	0.101	1.030	1.080	1.030	1.077
US-FR	−0.795	−0.939	0.718	0.097	1.033	1.028	1.028	1.030
FR-JP	−0.498	−0.891	1.000	−0.007	1.055	1.041	1.047	1.083
JP-FR	−1.269	−1.334	−0.467	0.021	1.043	1.038	1.026	1.036
UK-US	−0.681	−0.285	0.663	−0.055	1.021	1.025	1.058	1.083
US-UK	−0.456	−0.721	1.267	0.141	1.024	1.038	1.052	1.052
UK-JP	−0.855	−0.270	0.778	−0.015	1.018	1.016	1.048	1.080
JP-UK	−0.706	−0.640	−0.925	0.042	1.048	1.049	1.041	1.032
US-JP	−0.244	−0.744	0.516	0.068	1.088	1.069	1.058	1.080
JP-US	−1.426	−0.496	−2.118*	−0.011	1.034	1.050	1.070	1.033

Note: The pairs in panel B consist of a stock market (first country in each pair) and a bond market (second country in each pair), implying that one has to consider two stock-bond pairs for each pair of countries. Columns 1 and 2 report the W-test for the presence of stock-bond cocrashes (third data quadrant) or flight to quality (second data quadrant), respectively. The Z-statistic in column 3 reflects whether the difference between stock-bond contagion and flight to quality is statistically significant. * denotes statistical rejections at the 1% significance level. Columns 5–6 and 7–8 contain extremal linkage estimates reflecting the frequency of stock-bond cocrashes or flight to quality, respectively. The conditioning quantile pairs (expressed in percentages) are chosen in the vicinity of the historical sample boundaries.

column). For all other countries and market pairs the two effects are not significantly different from each other. On the other hand, the stock-bond correlations in table 3 are in most cases positive, which may be read as suggesting that cocrashes dominate flight-to-quality phenomena. This apparent difference between correlations and our extreme-events approach illustrates that full-sample correlations are unreliable as indicators for the direction of extreme comovements. One interpretation of these results on extreme cross-asset linkages (in particular the nonnegligible occurrence of FTQ) is that capital fleeing one market has to go somewhere else. This puts some bounds on how far contagious financial market crashes can reach.

The remaining columns report estimates of the conditionally expected number of simultaneous crashes in stocks and bonds (E_{CO}) and for flight to quality from stocks into bonds (E_{FTQ}). Apart from the now asymmetric conditioning, the extreme linkage results can be interpreted in the same manner as in the previous table. For example, for the pair consisting of France (FR) and the United Kingdom (UK) we expect that roughly 1 out of 20 ($\approx$1/0.052) stock crashes at the 20% level in France will coincide with an 8% decline in U.K. bonds. However, flight to quality from stocks into government bonds seems more likely for this country pair, because on average 1 out of 12.5 ($\approx$1/0.080) French stock crashes of 20% are expected to coincide with a U.K. bond market boom of 8%. For the reverse case (UK-FR) the figures mean that joint crashes in U.K. stocks and French bonds are more likely than a U.K. stock crash accompanied by a flight to quality into French bonds (6.8% > 5.1%). Just as in the previous table, the estimated extreme market linkage indicators are only slightly altered upon changing the conditioning quantile pairs.

In general, however, the expected values for cocrashes and flight-to-quality phenomena are found to be comparable in magnitude, that is, $E_{FTQ} \approx E_{CO}$, as already suggested by the Z-tests. Comparing the extreme linkage results in table 3 with the figures in table 2, we see that the linkage measures across different assets tend to be lower than those for linkages across bond markets, so that roughly the following ordering emanates from the data: $E^{SS}_{CO} > E^{BB}_{CO} > E^{SB}_{CO}$.[19]

The patterns for extreme cross-border linkages in table 3 (panel B) are surprisingly similar to within-country linkages (panel A). National borders do not seem to limit the degree of contagion or flight to quality, which illustrates well a potential disadvantage of international financial market integration from the perspective of domestic financial market stability. Also, it is very hard to disentangle any clear geographical patterns (for example, related to distance). For example, the three European countries FR, GE, UK do not seem to have systematically stronger extreme linkages with each other than with US or JP.

However, there is some evidence of safe-haven behavior. To see this compare the flight-to-quality effects for the country pairs involving the United States in the two cases XX-US and US-XX (XX = FR, GE, JP, or UK). For example, the probability that a government bond market rally in the United States coincides with a stock market crash in France is 7.7% (FR-US), whereas the reverse case in which the French bond market booms in case of a U.S. equity market crash (US-FR) has only a 3.0% probability. This relationship also holds for Germany and the United Kingdom (Japan being the only exception). It shows the historical role of U.S. government securities and the dollar as a safe haven for European investors.

VI. Conclusions

The linkages between asset markets in periods of crisis are characterized by their asymptotic tail dependence. From this measure we derive nonparametric estimates for the expected number of market crashes given that at least one market crashes. This novel approach does not rely on a specific probability law for the returns, and therefore has the distinct advantage over the often used conditional correlation measure that it cannot distort the view of the extreme spillover likelihood. Thus the approach in this paper bypasses the indirect method of computing a correlation and subsequently inferring the probability of loss, by directly calculating the economically relevant measure.

A first result for the G-5 countries from the univariate analysis is that stock market crashes on the order of a 20% weekly loss and government bond market crashes on the order of an 8% weekly loss are rare events, but nevertheless do happen once or twice per lifetime. Turning to the bivariate results, we found that stock markets experience a cocrash in approximately one out of five to eight crashes. This number is lower for bond markets, and tends to be still less for a cocrash between a stock and a bond market.

Nevertheless, returns of different G-5 securities markets seem to be statistically dependent during crises. In particular, following up on the upper bound 20% ballpark estimate for a cocrash between stock markets, given that for the United States, for example, a crash happens only approximately once every 40 years and considering that bond market or cross-asset cocrashes tend to happen less frequently, one may conclude that a widespread securities market meltdown in the main industrialized countries happens much less than once every 200 years. Also, the frequency of such phenomena among G-5 countries is much higher than what a normal-distribution-based analysis would one lead to believe. On the other hand, the flight-to-quality phenomenon is about as common as the cocrash of a bond and a stock market, highlighting some limits to the propagation of financial market crises across asset classes. And finally, whereas the likelihood that a securities market crisis reaches a certain breadth is of significant magnitude, it does not seem large in absolute terms. This point is even further strengthened by our finding that in small samples our linkage measure may have a bias that leads to an overestimation of the probability of extreme financial market spillovers. So, one implication of our analysis is that securities market contagion (a severe crisis in one market spilling over to another market) cannot be a prevalent phenomenon among G-5 countries. Overall, our results seem to be in line with some very recent literature arguing for other reasons that the financial market contagion phenomenon may have been overestimated in the earlier literature on financial market crises (see, for example, Forbes & Rigobon, 2002). This should not lead policymakers into complacency, for the next crisis might still be broad and be associated with contagion.

In line with free capital flows and financial integration between G-5 countries, national borders do not seem to matter very much. From the perspective of domestic financial stability this might sometimes be regarded as the downside of such integration, suggesting that the surveillance of financial market stability cannot stop at national borders.

[19] The extreme linkage results in tables 2 and 3 also have implications for portfolio VaR estimations very far into the tails of financial market return distributions. This is because the degree of extremal dependence determines the potential for diversifying risk. For a given low significance level, the VaR levels of pure stock portfolios will usually be higher than those of mixed (stock-bond) portfolios, whereas one may expect those of pure bond portfolios to take on an intermediate position. For readers interested in the details of the implications of our work for risk management these results are available upon request.

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APPENDIX A

Derivation of the Tail Dependence Coefficient

In this appendix we provide a heuristic derivation of the tail dependence coefficient η used in the Peng test introduced in section IV. Equation (19) implies for k, $n \to \infty$ and $t = k/n \to 0$

$$\frac{P\{X > Q_1(2k/n),\ Y > Q_2(2k/n)\}}{P\{X > Q_1(k/n),\ Y > Q_2(k/n)\}} \to 2^{1/\eta}. \tag{A-1}$$

Denote by $X_{i,n}$ and $Y_{i,n}$ the i^{th} ascending order statistics of the two return series. To estimate η from the sample we may replace P, Q_1, and Q_2 in equation (A-1) by their empirical counterparts. Therefore write

$$C_n(k, k) = k^{-1} \sum_{i=1}^{n} \mathbf{1}\{X_i > X_{n-k,n} \text{ and } Y_i > Y_{n-k,n}\}, \tag{A-2}$$

and substitute equation (A-2) into (A-1). Taking logs renders

$$\hat{\eta} = \ln 2/\ln\left(\frac{C_n(2k, 2k)}{C_n(k, k)}\right);$$

see Peng (1999), who also proved consistency and asymptotic normality of the estimator. Consequently the test statistic $W = (\hat{\eta} - 1)/\sigma(\hat{\eta})$ converges to a standard normal distribution and can be used to test the null hypothesis of asymptotic dependence ($H_0 : \eta = 1$) against the alternative hypothesis of asymptotic independence ($H_1 : \eta < 1$).

APPENDIX B

Small-Sample Properties of Tests

In this appendix we study the small-sample properties of the Peng pretest for asymptotic dependence [equation (20)] and of the Z-test [equation (18)]. A Monte Carlo study of the size and power properties for W requires choosing data-generating processes for simulating return pairs under H_0 and H_1. As H_0 we choose the bivariate Pareto distribution on $[1, \infty) \times [1, \infty)$,

$$F(x, y) = 1 + (x + y - 1)^{-\alpha} - x^{-\alpha} - y^{-\alpha},$$

with correlation coefficient $\rho = 1/\alpha$ (α being the tail index). Note the close association between tail dependence and tail fatness in this distribution. Asymptotically independent data are drawn from either a bivariate normal or a bivariate Gumbel-Pareto distribution. The latter distribution is

$$G(x, y) = (1 - x^{-\alpha})(1 - y^{-\alpha})[1+\gamma x^{-\alpha}y^{-\alpha}], \qquad 0 \leq \gamma \leq 1,$$

with correlation coefficient

$$\rho = \frac{\alpha(\alpha - 2)}{(2\alpha - 1)^2}\gamma.$$

Like the bivariate Pareto, the Gumbel-Pareto distribution has Pareto-distributed marginals. But unlike the bivariate Pareto distribution, it is asymptotically independent, because the tail dependence coefficient η equals ½. Similarly, the bivariate normal distribution is asymptotically independent, because $\eta = (1 + \rho)/2$. Interestingly, for the normal case the tail dependence parameter varies with the correlation coefficient, but η is constant for the Gumbel-Pareto distribution. These distribution functions represent a sufficiently rich dependence structure to evaluate the W-test performance.

In table B1 we report small-sample critical values for W under H_0 (bivariate Pareto distribution exhibiting asymptotic dependence) for different significance levels θ, degrees of tail dependence α, and threshold choices k. The values for α and k are chosen in accordance with the correlations observed in our data and the thresholds employed in the empirical section, respectively.

The table suggests that one should be careful using the normal critical values of -1.65 ($\theta = 5\%$) and -1.96 ($\theta = 2.5\%$) for testing asymptotic dependence in small samples. Assuming that the empirical financial data are also bivariate-Pareto-distributed, we can check whether the W-values from the empirical section lie within the small-sample rejection areas from table B1. More specifically, for each of the seventy asset pairs in tables 2 and 3 we selected the θ% rejection area for values of α and k that most closely resemble their true values, that is, the estimated correlation and the threshold choice for each asset pair.[20] Despite the tendency for lower critical values shown in table B1, we find that the empirical section's test results for asymptotic dependence hardly change.

Next we investigate the power properties of W. The simulation setup consists of three steps:

1. We draw samples of size $n = 663$ from the bivariate Pareto (H_0), the bivariate normal, and the bivariate Gumbel-Pareto (H_1) distribution and calculate η for the asymptotically dependent and independent samples. We then condition estimates of η on $k = 75$ and repeat this sampling scheme rep ($=10{,}000$) times. This renders the small-sample distributions $\tilde{\eta}$ (H_0) and $\tilde{\eta}^{ind}$ (H_1).
2. We estimate the 5% quantile of $\tilde{\eta}$ by the order statistic $\tilde{\eta}_{i,rep}$ with $i/rep = 0.05$.
3. We evaluate the small-sample power:

$$P_{SS} = \frac{1}{rep}\sum_{j=1}^{rep} \mathbf{1}\{\tilde{\eta}^{ind}_{j,rep} < \tilde{\eta}_{i,rep}\}.$$

[20] Because of space considerations we did not report threshold choices for each of the seventy empirical return pairs in tables 2 and 3. They are available upon request.

TABLE B1.—SMALL-SAMPLE CRITICAL VALUES OF THE W-TEST FOR BIVARIATE PARETO DRAWS

	$W(\theta = 5\%)$			$W(\theta = 2.5\%)$		
α	$k = 25$	50	75	25	50	75
2	−0.875	−1.274	−1.631	−0.925	−1.326	−1.693
3	−0.822	−1.244	−1.628	−0.867	−1.289	−1.675
4	−0.770	−1.204	−1.600	−0.818	−1.250	−1.645

Note: Critical values are determined for Peng's W-test for testing asymptotic dependence. The small sample df of W is obtained by 10,000 Monte Carlo replications from the bivariate Pareto df. The critical values are conditioned on different values of the significance level θ, the tail index α, and the threshold k of the tail dependence function. The range of k is consistent with the threshold values used in the empirical application.

TABLE B2.—SMALL-SAMPLE POWER OF THE W-TEST

	H_0 (Bivariate Pareto)		
	$\alpha = 2$	3	4
	H_1 (Normal)		
$\rho = 0$	86.94	73.49	60.52
$\rho = 0.25$	61.53	37.59	21.88
$\rho = 0.5$	21.62	5.26	1.15
	H_1 (Gumbel-P)		
$\gamma = 0.5$	87.64	67.72	50.95
$\gamma = 0.9$	86.68	63.1	42.29

Note: The power of the W-test is determined under the null hypothesis of a bivariate Pareto df and under two different alternative hypotheses with asymptotically independent tails. The small-sample dfs of W under asymptotic dependence and asymptotic independence are derived from 10,000 Monte Carlo replications. The alternative models might exhibit some statistical dependence reflected by the parameters ρ and γ, but this dependence dies out very far into the tails. Further details on the power calculations are provided in the main text.

Table B2 reports the power of W simulated by applying this procedure (for $k = 75$). The power clearly decreases in α, reflecting the degree of tail dependence for the null model (the stronger tail dependence under the null, the higher the power). Moreover the power under the Gumbel-Pareto alternative is nearly unaffected by changes in ρ because $\eta = 0.5$, regardless of the correlation value. As for the power under the H_1 of normality, it decreases when η (and thus ρ) is increased. This implies that, for any given sample size and significance level, a normal alternative with dependence parameter η close to 1 can be found that is indistinguishable from the null model of asymptotic dependence, that is, the *local alternatives* problem. However, Heffernan (2001) shows that a large majority of parametric models with asymptotically independent tails are governed by a tail dependence parameter of ½. Thus, one might interpret the power results under the normal alternative as the worst case to be encountered.

We also investigated the small-sample critical behavior of the Z-statistic equation (18) by applying the above simulation setup. Table B3 shows that small-sample critical values are again deviating from their asymptotic counterparts, but they are reasonably close to them for k-values of 50 or 75. Applying these alternative rejection regions to the Z-values in table 3 does not alter the test results. However, the Z-test may exhibit low power in small samples.

TABLE B3.—SMALL-SAMPLE CRITICAL VALUES OF THE Z-TEST FOR BIVARIATE PARETO DRAWS

	$Z(\theta = 5\%)$			$Z(\theta = 2.5\%)$		
α	$k = 25$	50	75	25	50	75
2	−1.438	−1.500	−1.564	−1.716	−1.807	−1.872
3	−1.395	−1.534	−1.554	−1.670	−1.869	−1.813
4	−1.426	−1.571	−1.551	−1.700	−1.867	−1.853

Note: Critical values are determined for the Z-test in equation (18) for testing equality of the cocrash and flight-to-quality effects between stocks and bonds. The small sample df of Z is obtained by 10,000 Monte Carlo replications from the bivariate Pareto df. The critical values are presented for different values of the significance level θ, the tail index α, and the threshold k of the tail dependence function.

APPENDIX C

Small-Sample Properties of the Estimator

The linkage estimator (16) can be shown to constitute an upper bound on the true amount of comovement in the crisis area. We show this both analytically and by simulation. Let us start with the analytic argument. Assume symmetric and equal marginal distribution functions such that $P\{X > x\} = P\{Y > x\} = p$, where x is some crash level. The expected number of cocrashes in the crisis area (1) can now be rewritten as:

$$E_p = \frac{2}{p^{-1}P\{X > Q_1(p) \text{ or } Y > Q_2(p)\}} \rightarrow \frac{2}{l(1, 1)} \quad \text{for } p \rightarrow 0, \tag{C-1}$$

and where $l(1, 1)$ is defined as in equation (3). This limiting relationship suggests the following estimator for E_p:

$$\hat{E} = \frac{2}{\hat{l}(1, 1)}, \tag{C-2}$$

with $\hat{l}(1, 1) = k^{-1} \sum_{i=1}^{n} \mathbf{1}(X_i > X_{n-k,n} \text{ or } Y_i > Y_{n-k,n})$. Notice the analogy with the proposed estimators in equations (15) and (16), the only difference being that we allowed for marginal asymmetry and inequality in the empirical section. It can now been shown that $\hat{E}$ exceeds E_p by applying Peng's finite-sample expansion for the bivariate tail (19) to the denominators of $\hat{E}$ and E_p.

To demonstrate this claim, note that the bivariate tail probability in the denominator of E_p is nested into Peng's expansion for $t = p$ and $u = v = 1$:

$$p^{-1}P\{X > Q_1(p) \text{ or } Y > Q_2(p)\} = 2 - c(1, 1)p^{\frac{1}{\eta}-1}[1 + O(p)^{\beta}]. \tag{C-3}$$

Likewise we obtain a finite-sample expansion for $l(1, 1)$ for $t = k/n$ and $u = v = 1$ in equation (19):

$$\hat{l}(1, 1) = 2 - c(1, 1)\left(\frac{k}{n}\right)^{\frac{1}{\eta}-1}\left[1 + O\left(\frac{k}{n}\right)^{\beta}\right]. \tag{C-4}$$

The choice of p reflects the crisis area we are interested in. In the empirical application we typically condition on crash levels Q_1 and Q_2 at the sample boundary or beyond, which corresponds to significance levels smaller than the reciprocal of the sample size ($p \leq n^{-1}$). It then immediately follows that equation (C-4) exceeds (C-3). Notice also that the second-order expansions lie closer to each other when the bivariate tail exhibits asymptotic dependence ($\eta = 1$). The small-sample bias in the linkage estimator should thus be lower under asymptotic dependence than under asymptotic independence.

We performed a Monte Carlo study of the small-sample bias, using a bivariate normal distribution (asymptotically independent tails) and a bivariate Pareto distribution (asymptotically dependent tails) as data-generating processes. Estimated linkage measures are reported in Table C1 for different sample sizes, threshold values k, and correlations ρ. The reported estimates are averages over 100 replications. Correlations and thresholds k are again chosen close to the observed correlations in the data and values of k used in the empirical section. The theoretical linkage value E_p is also recorded and evaluated at the sample boundary ($p = n^{-1}$).

The values of the measure provide an upper bound for the amount of linkage E_p during crisis periods. The amount of upward bias is higher in panel A, which is consistent with our analytical considerations on the bias. The difference between the asymptotically independent normal distribution and the bivariate Pareto distribution becomes apparent in the larger sample. For data sets 10 times larger than the current sample ($n = 663$), the normal-based linkage estimates start to approach 1 while the Pareto-based estimates retain their higher levels.

TABLE C1.—UPWARD BIAS IN LINKAGE ESTIMATORS: BIVARIATE NORMAL VERSUS BIVARIATE PARETO

	$n = 663$				$n = 5{,}000$			
	Est. $E[k]$				Est. $E[k]$			
ρ	$k = 25$	50	75	E_p	25	50	75	E_p
A. Bivariate Normal								
1/2	1.117	1.165	1.204	1.032	1.052	1.070	1.082	1.015
1/3	1.075	1.111	1.145	1.012	1.022	1.032	1.042	1.004
0	1.018	1.040	1.060	1.000	1.002	1.005	1.007	1.000
B. Bivariate Pareto								
1/2	1.173	1.198	1.219	1.149	1.152	1.156	1.162	1.145
1/3	1.120	1.148	1.167	1.081	1.086	1.096	1.103	1.073

Note: The table reports estimated values (Est. E) and "true" (analytic) values (E_p) of the extreme linkage measure for the bivariate normal and bivariate Pareto dfs and for different sample sizes n and correlations ρ. Moreover, the linkage estimates are presented for different values of the threshold k. The conditioning quantiles for the linkage estimates and analytic counterparts are chosen such that the corresponding marginal excess probabilities are equal to the inverse of the sample size, that is, exactly at the sample boundary.

APPENDIX D

Data Description and Discussion

Data were obtained from Datastream, Inc. G-5 countries are listed in Tables 1, 2, and 3 with the following abbreviations: France (FR), Germany (GE), United Kingdom (UK), United States (US), Japan (JP). The stock data are Financial Times/Standard & Poors world price indices, whereas the bond data correspond to price indices on 10-year (all-traded) government bonds. We did not include corporate bond indices, because of our particular interest in the flight-to-quality phenomenon. Returns were calculated as log price differences, Friday to Friday, in local currency. The stock and bond returns are not compensated for dividends and coupon payments, respectively. The sample of daily raw data used started on February 27, 1987 and ended on November 18, 1999, which amounts to 663 weekly observations.

Weekly data have the advantage that one significantly reduces the typical time zone problems encountered with international data at the daily frequency. Moreover, they capture more sustained crash phenomena, which can be expected to have more significant effects on financial institutions and the real economy than one would usually pick up with daily returns. An even longer holding period was not possible, due to the limited length of the bond index data available to us. Of course, the two advantages of the use of weekly data mentioned above come at the cost of not being able to address explicitly intraday or daily short-run dynamics that could also help to understand crisis propagation mechanisms. Although that is outside the scope of the present paper, in future work we plan also to investigate the time structure of extreme financial market spillovers by studying the following intertemporal specification of our extreme linkage indicator: $P(X_t > x, Y_{t+k} > y)$ for $k > 0$.