Journal of Empirical Finance 17 (2010) 241–254

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Heavy tails and currency crises[☆]

P. Hartmann[a], S. Straetmans[b], C.G. de Vries[c,*]

[a] European Central Bank and CEPR, DG Research, Kaiserstraße 29, 60311 Frankfurt, Germany
[b] Limburg Institute of Financial Economics (LIFE), Economics Faculty, Maastricht University, P.O.Box 616, 6200 MD Maastricht, The Netherlands
[c] Erasmus Universiteit Rotterdam, Economics Department, P. O. Box 1738, 3000 DR Rotterdam, The Netherlands

ARTICLE INFO

Available online 26 September 2009

JEL classification:
G1
F3
C49

Keywords:
Financial crises
Currency market linkages
Heavy tails
Convolutions
Asymptotic dependence

ABSTRACT

In affine models of foreign exchange rate returns, the nature of cross sectional interdependence in crisis periods hinges on the tail properties of the fundamentals' distribution. If the fundamentals exhibit thin tails like the normal distribution, the dependence vanishes asymptotically; while the dependence remains in the case of heavy tailed fundamentals as in case of the Student-*t* distribution. The linearity of the monetary model and heavy tail distributed fundamentals are sufficient conditions for fundamentals-based repeated joint currency crises. An estimator for the extreme exchange rate interdependencies is obtained and applied to Western, Asian and Latin American currency block data.

1. Introduction

In the year 1963 Mandelbrot published two remarkable papers (Mandelbrot, 1963a,b) in which he noted the two important features of speculative price data which gave financial econometricians enough to work on for the subsequent thirty years. The first feature is the Pareto nature of the tails of the distributions of financial returns. Heavy tails in itself were not new to economists, as Pareto himself had discovered this feature in high income data. But for financial economists, who had just discovered the niceties of Brownian motion and were on their way to option pricing, this was novel. In the end it carried the unfortunate message that markets are more likely to be incomplete than otherwise. The positive side of incompleteness is, though, that it gives sufficient motives for active hedging and risk management. The importance of this discovery only came to be fully recognized some thirty years later when banks started to calculate the Value-at-Risk (VaR) measure and had to come to terms with the non-normality of the distribution of financial asset returns. Due to the Pareto nature, the tail probabilities are self scaling, see Feller (1971, VIII.8). This intriguing mathematical phenomenon is very helpful for practical VaR calculations. Upon assuming that the self scaling property applied to the entire distribution, Mandelbrot concluded the distribution of asset returns had to be an infinite variance sum stable distribution.

The second empirical observation reported by Mandelbrot and attributed to Houthakker was the clustering of volatility feature. This data feature lay dormant until Engle's ARCH model (1982) gave a succinct representation which preserved the martingale property of asset prices. The ARCH model nicely implies that even though the innovations are normal, the stationary solution exhibits a Pareto type tail, consistent with the first data feature; see De Haan et al. (1989).

☆ The paper was delivered as a birthday present(-ation) at the Deutsche Bundesbank conference 'Heavy tails and stable Paretian distributions in finance and macro economics' organized in celebration of the 80th birthday of Professor Benoît Mandelbrot. The paper benefited from the insights of the discussant Mico Loretan and Dirk Tasche. We also would like to thank seminar participants in Bath and Maastricht. Finally, we are grateful to the co-editor of the Journal of Empirical Finance and two anonymous referees for their constructive comments.

* Corresponding author.
E-mail addresses: philipp.hartmann@ecb.int (P. Hartmann), s.straetmans@maastrichtuniversity.nl (S. Straetmans), cdevries@ese.eur.nl (C.G. de Vries).

0927-5398/$ – see front matter © 2009 Published by Elsevier B.V.
doi:10.1016/j.jempfin.2009.09.004

The 1963 articles also ask the question about the economic mechanism that could be responsible for the noted data features. As of today little work has been devoted to this question, although some early work on income distribution, notably Chapernowne (1953) and some other more recent work, do provide some insight for the marginal distributions to be fat tailed, see e.g. Gabaix et al. (2003). In this article we like to extend the original work of Mandelbrot and subsequent developments by discussing the economics and econometrics of extremes in a multivariate setting. Thirty years later we know a lot more about the univariate issues concerning the heavy tail and summability features. For example, we now realize it suffices to assume the scaling only holds in the tail area. This has the benefit that one does not need to make the infinite variance assumption. But the multivariate issues are wide open for research. The aim of the paper is to show in which direction research is currently evolving.

For this purpose we analyze the multivariate dependency structure in affine models of foreign exchange rate returns. We clarify the cross sectional return dependency during crises by combining the univariate data feature discovered by Mandelbrot with Frenkel's monetary model of the exchange rate. In doing so we do not explain the Pareto tail nature of the univariate series, which we take as given, but we offer an economic explanation for the observed strong crisis spillovers. While fat tails and tail dependence of forex returns have by now been extensively documented in the empirical literature, how the marginal tail thickness relates theoretically to the bivariate and multivariate tail dependence of returns in standard foreign exchange rate models has not been dealt with before. The multivariate questions are certainly new and could hardly have been addressed at the time Mandelbrot was writing his fascinating articles.

There is a long standing discussion about the origins of financial crises. One view holds that such crises are the expression of an occasional inherent malfunctioning of financial institutions or markets (sunspots). Another view rather sees crises as caused by bad outcomes in underlying economic variables (fundamentals). Representative of the first view is the literature modelling univariate crises as self-fulfilling events in the presence of multiple equilibria. For example, Diamond and Dybvig (1983) show that bank depositor runs can occur as a self-fulfilling prophecy, which would imply that they happen more or less randomly. Obstfeld (1986) argues that currency crises can occur as a consequence of multiple equilibria. This is in contrast with the literature which links crises to unfavorable macroeconomic conditions, sometimes caused by bad economic policies. For example, Gorton (1988) argues that most episodes of banking instability in US history were related to business cycle downturns. Krugman (1979) shows how unsustainable large budget deficits can lead to currency attacks. According to Gencay and Selcuk (2006) domestic financial repression may also be an important trigger of emerging currency crashes.[1] Masson (1999) nests most of the previously mentioned channels in a model of contagious currency crises.

In the present paper we do not take a position on the two views of self-fulfilling and fundamentals-based exchange rate crises. Rather, we concentrate on the significance of a crisis. In particular we give sufficient conditions for the repeated occurrence of widespread crises. We use joint currency crashes as an application to illustrate our point, but the argument is more general. It applies to any group of assets whose values are linearly driven by underlying economic variables, or risk factors.

Two basic conditions are sufficient for the frequent occurrence of systemic (widespread) market crises. First, the univariate distributions describing the behavior of economic variables (or 'fundamentals') underlying the exchange rates need to exhibit heavy tails. Loosely speaking, the heavy tail feature means that the probability of univariate currency collapses is approximately Pareto distributed. Mandelbrot's hypothesis of non-normal stable distributions and Student-*t* distributions exhibit an expansion of their tails in which the first term is a power function like Pareto's distribution. This implies that the probability of a currency crisis is much higher than one would expect if the underlying fundamentals were normally distributed (with the same mean and variance). Second, the (logarithm of) nominal bilateral exchange rates, expressed against the same base currency, are linear expressions of the domestic and the base currency fundamentals. The standard monetary model of the foreign exchange rate provides such an affine framework. The two conditions are shown to imply that joint currency crises will occur frequently and with vehemence. The surprising element of this result is that the degree of cross-sectional dependence between exchange rate returns during crisis periods (so called asymptotic dependence) is related to the univariate characteristics of the tails of the fundamentals' distributions.[2]

One may therefore classify currency linkages during times of market stress into a weak and a strong type, depending on whether the conditional crash probability vanishes or remains asymptotically. Correspondingly, the international monetary and financial system may be characterized as being relatively stable in the former case, while it is more fragile in the latter case. Our two conditions, univariate heavy tails and linearity, are sufficient for a more fragile system. We emphasize that in general the dependency structure of a multivariate distribution and the shape of the univariate distributions are two unrelated concepts. But here the affine economic structure induces that the characteristics of the marginals affect the multivariate dependency structure in a specific way. The same also applies to other asset classes. For example, bank equity returns are asymptotically dependent, if banks hold correlated linear portfolios of partly common, heavy-tail distributed assets, see De Vries (2005).[3]

In the empirical application of the paper, we assess the strength of the asymptotic dependence for panels of industrial (developed) and emerging market currencies by using a simple count based estimator for the extreme interdependencies.[4] The

[1] Gencay and Selcuk also argue that the 2001 Turkish currency crisis could have been expected on the basis of the pre-crisis extreme value distribution of overnight interbank rates.

[2] The cross sectional dependence remains in the tail areas, if the forex fundamentals follow infinite variance sum stable distributions as suggested by Mandelbrot (1963a,b). But we argue that the strong dependence is a more general phenomenon, as it applies to all regularly varying (Pareto type tails) distributed fundamentals.

[3] For a broad survey of the contagion literature based on correlation analysis, see De Bandt and Hartmann (2000).

[4] Recently, a number of studies employed multivariate extreme value analysis to measure extreme asset return linkages, see Hartmann et al. (2004) and Poon et al. (2004). Related bivariate analyses for a single type of asset have recently been carried out on foreign exchange data by Starica (1999) and Hartmann et al. (2003).

linkage estimates are suggestive of considerable asymptotic dependence. The amount of extreme dependency that we find varies, however, with the choice of the numeraire currency. We argue that this heterogeneity stems from the scale magnitudes of the fundamental variables which drive the base currency.

We see two policy implications. One is that risk management has to take into account that the likelihood of multiple currency crises is much higher than under the usual multivariate normal distribution assumption. The second implication is that macro policies instead of micro market quirks are the main drivers of the larger exchange rate swings. This implies that prudent (monetary) policy should exhibit moderation instead of overshooting and feeding onto the market uncertainties.

The remainder of this paper proceeds as follows. In Section 2 we introduce the canonical affine exchange rate model in which we study the relationship between marginal tail thickness and bivariate tail dependence. A discussion and comparison of different measures to characterize currency linkages during periods of market stress is provided in Section 3. The central result of the paper on the relationship between the exchange rates' marginal tail properties and the degree of tail dependence is in Section 4. The two cases of thin tailed and fat tailed marginals are treated in two separate subsections. Section 5 discusses estimators and presents empirical evidence. Section 6 concludes. The Appendix contains a number of insightful figures.

2. Affine forex models

Consider the standard monetary model of the log price of currency j per currency 0

$$\begin{aligned} s_{j0} &= (m_j - \phi y_j + \lambda R_j) - (m_0 - \phi y_0 + \lambda R_0) \\ &= g_j - g_0, \quad j = 1, \cdots, n. \end{aligned}$$

Here g_j and g_0 are composite fundamentals consisting of the logarithmic money measure m, the negative of the income elasticity times log real income $-\phi y$ and the semi interest rate elasticity times the interest rate λR, see e.g. Frenkel (1976). In first differences the monetary model can be concisely summarized as

$$\Delta s_{j0} = \Delta g_j - \Delta g_0. \tag{1}$$

The linear in first difference specification reveals two properties that will prove crucial in the following sections. First, the set of multiple exchange rate returns Δs_{j0} ($j = 1, \cdots, n$) all have the fundamental Δg_0 in common. This exposure to shocks in the numeraire currency may be important, as illustrated e.g. in Aghion et al. (2001). For a set of emerging market currencies, these authors plot the ratio of dollar denominated liabilities to claims with respect to foreign banks in 1997 right before the onset of the Asian crisis.[5] Given the high content of dollar denominated debt, most of the emerging market currencies were therefore highly exposed to the same US interest rate fluctuations. Second, Eq. (1) is linear in the first differences of the composite fundamental g and hence linear in the first differences of the individual fundamentals as well. The linear specification conforms e.g. to the linear factor model used in Forbes and Chinn (2003), who show that trade linkages are important transmitters of shocks between countries.[6] The use of linear models is by no means limited to the monetary model or the exchange rate literature, cf. the popular Arbitrage Pricing Theory for explaining equilibrium equity returns (Ross, 1976). Thus our results pertain to linkages between other classes of assets as well.

3. Measures of dependency

A standard measure of dependency is the coefficient of correlation ρ. However, correlations can be quite misleading indicators of dependence during crisis episodes as the concept is so intimately related to the multivariate normal distribution. For example, Ang and Chen (2002) demonstrate for the bivariate normal distribution that the correlation varies considerably when truncated (i.e., defined over a subset of returns) and eventually goes to zero in case of two-variable truncation in the bivariate tail. In addition, the truncated correlation differs across different classes of multivariate distributions.[7] Finally, economists evaluating investments within an expected utility theory framework are not so much interested in the correlation measure itself; they rather have an interest in the trade-offs between risk measured as a probability and the gains or losses, which are the quantiles of the return distribution. As such, the correlation is only an intermediate step in the calculation of this trade-off between quantile and probability. Therefore we like to turn to a measure which is not conditioned on a particular multivariate distribution and which directly reflects the probabilities and associated crash levels. For a more in-depth treatment of the pitfalls of correlation analysis, see e.g. Embrechts et al. (1999).

What is worrying for supervisors and industry representatives is that a heavy loss in one market goes hand in hand with a heavy loss in another market, destroying the real value of a diversified investment portfolio. More specifically, one asks given that $Y > s$, what is the probability that $X > s$, where X and Y stand for currency returns and s is the common high loss level.[8] Since we are interested in the extreme linkage probabilities, we will try to directly evaluate these probabilities, bypassing the correlation concept.

[5] The ten most highly exposed countries are found to be Thailand, Indonesia, Russia, Korea, Malaysia, South Africa, Philippines, Colombia, Mexico and Brazil, respectively.

[6] Note that the monetary model captures the mirror image of the trade account through movements in the capital account.

[7] The correlation is also only a meaningful concept if the second moments are bounded.

[8] Without loss of generality we can take the two quantiles on which we condition equal to s.

If two random variables X and Y are not independent, having some information about one variable, say X, implies that one has also information about the other variable, Y. This can be readily expressed as a conditional probability $P\{Y>s|X>s\}$. We will, however, adopt the related probability measure that conditions on any market crash, without indicating the specific market. More specifically, let κ denote the number of simultaneously crashing currencies, i.e. all currency returns exceeding s. Denote the conditionally expected number of currency crashes, given a collapse of at least one currency, as $E\{\kappa|\kappa\geq 1\}$. From elementary probability theory we have that

$$E\{\kappa|\kappa \geq 1\} = 1\frac{P\{X>s, Y\leq s\} + P\{X\leq s, Y>s\}}{1-P\{X\leq s, Y\leq s\}} + 2\frac{P\{X>s, Y>s\}}{1-P\{X\leq s, Y\leq s\}} = \frac{P\{X>s\} + P\{Y>s\}}{1-P\{X\leq s, Y\leq s\}}, \tag{2}$$

as proposed in Huang (1992) and employed by Hartmann et al. (2004).

The conditional expectation measure $E\{\kappa|\kappa\geq 1\}$ has the advantage that it can be easily extended beyond the bivariate setting.[9] Moreover, one does not need to specify the conditioning crashing currency whereby one would look only into one direction of the plane.

To develop some intuition for this measure as a device for measuring dependence during times of market stress, consider two polar cases.

Case 1. If X and Y are independent and identically distributed (i.i.d.) and writing $p=P\{X>s\}$, then

$$E\{\kappa|\kappa\geq 1\} = \frac{2p}{1-(1-p)^2} = \frac{2}{2-p}.$$

In the limit $p\to 0$ as $s\to\infty$, and hence $E\{\kappa|\kappa\geq 1\}\to 1$.

Case 2. If $Y=a+bX$ ($b\neq 0$) and writing $p=P\{X>s\}$, then

$$E\{\kappa|\kappa\geq 1\} = \frac{2p}{1-(1-p)} = 2.$$

Clearly, even as $p\to 0$, still $E\{\kappa|\kappa\geq 1\}=2$.

These two cases show that $1\leq E\{\kappa|\kappa\geq 1\}\leq 2$. In case the return pair is completely independent, $E\{\kappa|\kappa\geq 1\}$ reaches its lower bound for very large quantiles s, which implies that the data are also asymptotically independent.[10] On the other hand, if the data are completely dependent, then in the limit ($s\to\infty$), $E\{\kappa|\kappa\geq 1\}$ will equal 2 (complete asymptotic dependence). Also notice that even though in the first case the data are independent, the dependency measure $E\{\kappa|\kappa\geq 1\}$ is higher than 1 at all finite levels of p since even with independent returns, there is a nonzero probability that 'two markets will crash, given that at least one market crashes'. As for the intermediate case of imperfectly correlated returns ($\rho\neq 0$, $|\rho|<1$), either $E\{\kappa|\kappa\geq 1\}=1$ (asymptotic independence) or $1<E\{\kappa|\kappa\geq 1\}\leq 2$ (asymptotic dependence) may hold, if the quantile s gets large. Notice that asymptotically dependent currency returns are most of the time imperfectly comoving in relevant empirical applications, i.e. $1<E<2$. Also, one cannot rule out that currency returns are asymptotically independent in the presence of a nonzero correlation. An example is discussed in the next section.

One might be tempted to use a certain Value at Risk level used to evaluate market risk. But it so happens that these VaR levels are usually taken so close to the center that they are uninformative about stress situations (at the 5% level per one hundred trading days, i.e. a frequency of once per twenty days). Since economic theory does not say which level constitutes a stress level, we evaluate the conditional expectation in the limit, i.e. use $\lim_{s\to\infty} E\{\kappa|\kappa\geq 1\}$. From statistical extreme value theory we know that even if we evaluate Eq. (2) in the limit, this still provides a good approximation of what one can expect at high but finite levels, see e.g. Balkema and De Haan (1974).

4. Weak and strong crisis linkages

Within the affine currency model framework from Section 2, we are now ready to prove that the limiting value of Eq. (2) as $s\to\infty$ critically depends on the tail properties of the marginal distributions of the currency fundamentals. We classify the crisis linkage as weak (asymptotic independence) whenever $E\{\kappa|\kappa\geq 1\}=1$ in the limit, and strong (asymptotic dependence) otherwise. If the former case applies, the international monetary and financial system is more stable as severe crises in one currency are unlikely to be associated with crises in other currencies. In the latter case systemic risk is present and the system is potentially fragile.

Assume that each of the countries' composite fundamentals Δg in Eq. (1) is independent from all the other countries' composite fundamentals. Regarding the distribution of Δg, we either assume normality or that the distribution exhibits heavy tails in the sense that tail probabilities are declining as a power function of the quantile (to be made precise below). Notice that tails of the normal distribution are governed by the exponential function whereas heavy tailed probability laws like the non-normal stable distributions or the Student-t exhibit a Pareto distribution-type decline. It is nowadays an accepted stylized fact that many asset returns are heavy tailed. We show that this necessarily leads to asymptotic dependence. Conversely, we also show that if the fundamentals exhibit light tails, such as the normal distribution, then the forex returns are asymptotically independent.

[9] Within the bivariate setting it holds that $E\{\kappa|\kappa\geq 1\}=P\{\kappa=2|\kappa\geq 1\}+1$; but this relation breaks down in higher dimensions. Moreover, and as we will illustrate in the empirical application, the estimation of $E\{\kappa|\kappa\geq 1\}$ in higher dimensions is straightforward in contrast to the probability $P\{\kappa=N|\kappa\geq 1\}$.

[10] "Complete" independence of (X, Y) refers to independence over the full support of the joint distribution (thus not only in the tail area). It follows that complete independence is sufficient for tail independence but not vice versa.

In order to derive our main result it is sufficient to consider a three currency system with composite fundamentals $\Delta g_0 = -X$, $\Delta g_1 = Y$, and $\Delta g_2 = Z$ such that $\Delta s_{10} = Y + X$ and $\Delta s_{20} = Z + X$. We assume that the risk factors X, Y, and Z are i.i.d. [11]

4.1. Fundamentals with light tails

In this subsection we assume that X, Y and Z are standard normally distributed random variables. As normality is preserved under summation the pair of random variables (Δs_{10}, Δs_{20}) exhibits a bivariate normal distribution with correlation coefficient $\rho = 1/2$. Sibuya (1960) first proved that a bivariate normal distribution exhibits asymptotic independence, but in the proof below we use finer expansions rather than the bounds.

Proposition 1. *If Δs_{10} and Δs_{20} follow a bivariate normal distribution with $\rho = 1/2$, then* $\lim_{s\to\infty} E(\{\kappa|\kappa \geq 1\} = 1$, *so that the crisis linkage is weak.*

Proof. We start by noticing that the linkage measure Eq. (2) can be transformed as follows:

$$E\{\kappa|\kappa \geq 1\} = \frac{P\{\Delta s_{10} > s\} + P\{\Delta s_{20} > s\}}{1 - P\{\Delta s_{10} \leq s, \Delta s_{20} \leq s\}} = \frac{1}{1 - \frac{P\{\Delta s_{10} > s, \Delta s_{20} > s\}}{P\{\Delta s_{10} > s\} + P\{\Delta s_{20} > s\}}}. \tag{3}$$

To evaluate these probabilities, we use the expansions from Ruben (1964). The marginal tail probabilities in Eq. (3) are governed by the asymptotic expansion:

$$P\{\Delta s_{10} > s\} = P\{X + Y > s\} = P\left\{\sqrt{2}X > s\right\} \sim \frac{1}{\sqrt{\pi}}\frac{1}{s}e^{-s^2/4}$$

for large s. As for the joint failure probability in (4.1) we have

$$P\{\Delta s_{10} > s, \Delta s_{20} > s\} \sim \frac{9}{2\pi\sqrt{3}}\frac{1}{s^2}e^{-s^2/3}.$$

Thus, upon combining the last expressions and under the stated normality assumptions

$$\lim_{s\to\infty} \frac{P\{\Delta s_{10} > s, \Delta s_{20} > s\}}{2P\{\Delta s_{10} > s\}} = \frac{9}{4\sqrt{3\pi}} \lim_{s\to\infty} \frac{1}{s} e^{-s^2/12} = 0.$$

Hence,

$$\lim_{s\to\infty} E\{\kappa|\kappa \geq 1\} = 1. \qquad \square$$

This asymptotic independence result is by no means limited to the class of normal distributions. A similar procedure can be used to verify the asymptotic independence for other types of distributions, see e.g. de Vries (2005). But the normal distribution is the most interesting distribution, since it is so often used in theoretical and empirical work on exchange rate returns and in other asset pricing applications. Note that we have just shown that this assumption implies that currency (or other financial market) contagion cannot occur systemically.

4.2. Fundamentals with heavy tails

Prior to relating the tail fatness of exchange rate fundamentals to their degree of asymptotic dependence, we need a formal definition of what the 'Pareto tails' exactly means. A random variable exhibits heavy tails if its distribution function $F(s)$ far into the tails has a first order term identical to the Pareto distribution, i.e.

$$F(s) = 1 - s^{-\alpha}L(s) \text{ as } s \to \infty, \tag{4}$$

where $L(s)$ is a slowly varying function such that

$$\lim_{t\to\infty} \frac{L(ts)}{L(t)} = 1, \; s > 0. \tag{5}$$

[11] In practice, basic fundamentals like money supplies, national income levels and interest rates cannot be considered as being independent across countries. However, it can be easily shown that the relationship we derive between marginal tail heaviness and bivariate tail dependence still holds for pairwise dependent X, Y and Z. The dependence – if present – actually even strengthens our results. By assuming independence we isolate the most difficult case to prove.

It can be easily shown that conditions (4)–(5) are equivalent to

$$\lim_{t\to\infty}\frac{1-F(ts)}{1-F(t)} = s^{-\alpha},\ \alpha>0, s>0, \tag{6}$$

i.e., the distribution varies regularly at infinity. The tail index α can be interpreted as the number of bounded distributional moments. And as not all moments are bounded, we speak of heavy tails. Distributions like the Student-*t*, *F*-distribution, Burr distribution, sum stable distributions with unbounded variance all fall into this class. It can be shown that the unconditional distribution of the ARCH and GARCH processes also belongs to this class, see De Haan et al. (1989) for a proof. Note that Student-*t* distributions are often used in the empirical modelling of the unconditional return of exchange rates, see e.g. Boothe and Glassmann (1987), while GARCH process are extremely popular conditional models.

To derive our result, we need Feller's convolution theorem (Feller, 1971, VIII.8):

Theorem 1. *Let X_i be i.i.d. random variables with regularly varying symmetric tails, i.e. as $s\to\infty$*

$$P\{X_i\le -s\} = P\{X_i>s\} = s^{-\alpha}L(s).$$

Then for the tail of the distribution of the sum of X_i $(i=1, N)$ (N-fold convolution) as $s\to\infty$

$$P\left\{\sum_{i=1}^{N} X_i\le s\right\}/\,1-Ns^{-\alpha}L(s) = 1. \tag{7}$$

Returning to the original Mandelbrot articles, recall that the non-normal sum stable laws were proposed since the data seemed to display the self scaling property. In particular, if the X_i are i.i.d. sum stable distributed with characteristic exponent $\alpha<2$, then

$$P\left\{\sum_{i=1}^{N} a_iX_i\le s\right\} = P\left\{\left[\sum_{i=1}^{N} a_i^{\alpha}\right]^{1/\alpha} X_i\le s\right\}, \text{for } a_i\ge 0. \tag{8}$$

Here we do not require sum stability for the entire distribution, but only require the tails of the distribution to exhibit the power like behavior which is so characteristic for the non-normal stable distribution. This corresponds better to reality. Feller's theorem then shows that the self scaling is nevertheless preserved in the tail area under this much weaker condition. Comparing Eqs. (7) and (8) shows that the scaling factor inside Eq. (8), will only show up in the first order term of the expansion Eq. (7). The Feller theorem informally says that to a first order in the tail area all mass is concentrated along the axes. This intuition implies the following:

Proposition 2. *Let X, Y and Z be i.i.d. random variables with regularly varying tails, i.e. as $s\to\infty$*

$$P\{X\le -s\} = P\{Y\le -s\} = P\{Z\le -s\} = s^{-\alpha}L(s), \tag{9}$$

$$P\{X>s\} = P\{Y>s\} = P\{Z>s\} = s^{-\alpha}L(s). \tag{10}$$

Then if $\Delta s_{10}=X+Y$ and $\Delta s_{20}=X+Z$, it holds that

$$\lim_{s\to\infty} E\{\kappa|\kappa\ge 1\} = \frac{4}{3}.$$

Proof. By definition

$$\lim_{s\to\infty} E\{\kappa|\kappa\ge 1\} = \lim_{s\to\infty}\frac{P\{\Delta s_{10}>s\}+P\{\Delta s_{20}>s\}}{1-P\{\Delta s_{10}\le s,\Delta s_{20}\le s\}} = \lim_{s\to\infty}\frac{P\{X+Y>s\}+P\{X+Z>s\}}{1-P\{X+Y\le s, X+Z\le s\}}. \tag{11}$$

By Feller's convolution Theorem 1 we directly have for the numerator in Eq. (11) that

$$P\{X+Y>s\}+P\{X+Z>s\}\sim 2s^{-\alpha}L(s)+2s^{-\alpha}L(s).$$

For the denominator

$$1-P\{X+Y\le s, X+Z\le s\},$$

note that the lines $X+Y=s$ and $X+Z=s$ are two of the three edges of the triangular plane $X+Y+Z=s$. We noted above that Feller's theorem implies that for large s all mass is along the axes. Hence, if we are interested in the joint probability of being below any two of the three edges of the triangular plane, this is necessarily equal to the probability of being below the triangular plane, since the set of two edges cuts the three axes at the same points as the triangular plane and only the mass along the axes counts (to a first order). For the complement of this probability, one counts the mass along the axes outside the triangular plane. Hence,

$$1-P\{X+Y\leq s,X+Z\leq s\}\sim 1-P\{X+Y+Z\leq s\}\sim 3s^{-\alpha}L(s).$$

Thus

$$\lim_{s\to\infty}\frac{P\{X+Y>s\}+P\{X+Z>s\}}{1-P\{X+Y\leq s,X+Z\leq s\}}=\lim_{s\to\infty}\frac{2s^{-\alpha}L(s)+2s^{-\alpha}L(s)}{3s^{-\alpha}L(s)}=\frac{4}{3}. \tag{12}$$

□

The two exchange rates returns Δs_{10} and Δs_{20} are asymptotically dependent, since $\lim_{s\to\infty}E\{\kappa|\kappa\geq 1\}=4/3>1$. Thus the crisis linkage for this class of distributions is strong and the international monetary and financial system appears relatively fragile, exhibiting systemic risk. For the sake of the argument the risk factors in the above were taken identical. One can easily allow for differences in scale, however, as long as the tail exponents are equal. Consider positive weights a, b, and c, such that

$$\Delta s_{10}=aX+bY,\ \Delta s_{20}=aX+cZ, \tag{13}$$

and where Eqs. (9) and (10) hold. Note, for example, that $P\{aX>s\}=a^{\alpha}s^{-\alpha}L(s)$ if Eq. (10) applies. Thus in the case of Eq. (13) one shows that Eq. (12) becomes

$$\lim_{s\to\infty}\frac{P\{\Delta s_{10}>s\}+P\{\Delta s_{20}>s\}}{1-P\{\Delta s_{10}\leq s,\Delta s_{20}\leq s\}}=1+\frac{a^{\alpha}}{a^{\alpha}+b^{\alpha}+c^{\alpha}}. \tag{14}$$

The amount of asymptotic dependence is determined by the relative sizes of the weights.

Following Slijkerman (2007, ch.4), the above result Eq. (14) for currency pairs can easily be generalized to currency blocks consisting of $n+1$ currencies (including the numeraire currency):

Corollary 1. *Suppose $\Delta s_{i0}=aX+b_iY_i$, where the X and Y_i are i.i.d. and have Pareto tails as in* Eqs. (9) and (10) *and positive weights a, b_i. Then*

$$\lim_{s\to\infty}\frac{\sum_{i=1}^{n}P\{\Delta s_{i0}>s\}}{1-P\{\Delta s_{10}\leq s,\ldots,\Delta s_{n0}\leq s\}}=1+\frac{(n-1)a^{\alpha}}{a^{\alpha}+\sum_{i=1}^{n}b_i^{\alpha}}. \tag{15}$$

Slijkerman (2007) analyzes the interdependencies between the equity returns of a large number of financials assuming the CAPM applies under the same distributional assumptions of market and idiosyncratic risk as we make for X and Y_i. He shows that the relative scale of the market risk versus idiosyncratic risk determines whether the linkage measure is close to 1 (i.e. asymptotic independence) or is much larger. Similarly, Corollary 1 shows that the linkage measure will approximately equal $2n/(n+1)$ when the scales of the domestic and foreign fundamentals are of similar order of magnitude. This situation may be typical for countries with a high degree of monetary integration. The linkage measure will be higher when the base currency fundamentals have scales which dominate the domestic fundamentals. This can arise if the base currency follows a highly inflationary and volatile monetary policy. The opposite occurs if the base currency weight a is small compared to the b_i, i.e. in case the base currency has relatively stable fundamentals.

5. Estimators and empirical evidence

In this section we develop an estimator for the expectation measure Eq. (2) and its multivariate equivalent and provide estimates of this measure using currency data.

5.1. Estimation theory

Consider the multivariate generalization of the expectation measure Eq. (2) for n exchange rate returns above a high threshold s, as in Eq. (15)

$$E\{\kappa|\kappa\geq 1\}=\frac{\sum_{i=1}^{n}P\{\Delta s_{i0}>s\}}{1-P\{\Delta s_{10}\leq s,\ldots,\Delta s_{n0}\leq s\}}=\frac{\sum_{i=1}^{n}P\{\Delta s_{i0}>s\}}{P\{max[\Delta s_{10},\ldots,\Delta s_{n0}]>s\}}. \tag{16}$$

Under the null of independently heavy tail distributed fundamentals that enter Δs_{i0} linearly, e.g. as in Eq. (12), the marginal probabilities in the numerator and the denominator all satisfy Eq. (4), such that

$$\lim_{s\to\infty} \frac{\sum_{i=1}^{n} P\{\Delta s_{i0} > s\}}{P\{max[\Delta s_{10}, ..., \Delta s_{n0}] > s\}} = C$$

exists for some constant $C \in [1, n]$.

Note that $max[\Delta s_{10}, ..., \Delta s_{n0}]$ is a single univariate random variable that varies regularly at infinity. The univariate tail probabilities in the numerator and the multivariate probability in the denominator can therefore be estimated by using univariate techniques. To this end we employ the asymptotically normally distributed estimator

$$\hat{p}_s = \frac{M}{T}\left(\frac{t}{s}\right)^{\hat{\alpha}} \tag{17}$$

from De Haan et al. (1994). In this expression s stands for the extreme quantile for which one desires the associated probability p_s, M is the number of observations (highest order statistics) above the threshold t, T is the number of observations in the sample and $\hat{\alpha}$ is an estimator of the tail exponent, such as the Hill estimator. The idea behind the semi-parametric estimator Eq. (17) is that the empirical tail probability M/T at quantile t, where it is still a reliable estimate since it is well inside the range of observations, is extended to the part where there are insufficient number of observations, by using the shape of the Pareto tail.

Since all the probabilities in the expectation measure have the same Pareto tail up to a constant, we substitute the de Haan et al. estimator for the marginal probabilities in Eq. (16) in order to obtain

$$\hat{E}\{\kappa|\kappa \geq 1\} = \frac{\sum_{i=1}^{n} \frac{M_i}{T}\left(\frac{t}{s}\right)^{\hat{\alpha}}}{\frac{M_{max}}{T}\left(\frac{t}{s}\right)^{\hat{\alpha}}} = \frac{\sum_{i=1}^{n} M_i}{M_{max}}, \tag{18}$$

and where M_i is the number of order statistics above t for the $\{\Delta s_{i0}\}$ series and $M_{\max}$ is the corresponding number for the $\{\max[\Delta s_{10}, ..., \Delta s_{n0}]\}$ sequence. The estimator is further simplified by taking the common cutoff point t in estimating the probabilities in the numerator and the denominator. It implies, inter alia, that once $M_{\max}$ is chosen all nuisance parameters in the numerator are automatically identified via t. The estimator thus becomes a simple count measure since the Pareto tail shape correction cancels from the numerator and the denominator. This implies that we do not need to estimate the tail index α separately.

The behavior of the linkage estimator Eq. (18) at different in sample threshold levels t is shown graphically in the Appendix for simulated data of correlated normal and correlated Student-t random variables and for one of the currency pairs of exchange rate data. Close to the maxima from the two series (at high threshold levels t) there are so few excesses that the plot is rather unstable, while it settles at a stable plateau in case of asymptotic dependence. In the end the plot traverses all the way from 1 to 2 once t is lowered until all observations are included. The difference between asymptotically dependent and independent data is that in the former case not far from the highest realizations a stable plateau emerges, while under asymptotic independence the plot at first does not leave the neighborhood of 1 and then slowly starts to rise towards 2. Note that the estimator in Eq. (18) is closely related to the stable tail dependence function derived in Huang (1992). This latter estimator fixes the marginal probability levels instead of fixing the threshold levels (quantiles) as in Eq. (18). The ratio of the sum of the marginal probabilities divided by the stable tail dependence function gives Eq. (2) for $s \to \infty$. Since Huang (1992) has shown that the stable tail dependence function estimator is asymptotically normally distributed, so is the ratio of the marginal probability sum divided by the stable tail dependence function, as well as the estimator that we use in the current paper.

In the application, we like to determine whether cross-sectional differences in the linkage measure between various groups of currencies, say between Asia, Latin America and the industrialized world, are statistically and economically significant. The asymptotic normality of $\hat{E}$ enables some straightforward hypothesis testing. A test for the equality of the conditionally expected number of currency crises across currency pairs or bigger currency blocks (null hypothesis) can be based on the following T-statistic:

$$T_{eq} = \frac{\hat{E}_1 - \hat{E}_2}{s.e.(\hat{E}_1 - \hat{E}_2)}, \tag{19}$$

which converges to a standard normal distribution in large samples. In the empirical application below the asymptotic standard error in the test's denominator Eq. (19) is estimated using a block bootstrap.[12]

For practical purposes, one somehow has to select a threshold t to determine the M_i and $M_{\max}$. To this end we plotted $\hat{E}$ as a function of $M_{\max}$ and selected the latter parameter in a stable region.[13] More sophisticated double bootstrap techniques have been developed (see Danielsson et al., 2001) but these are only advisable for sample sizes that are typically much larger than in this paper.

[12] The blocks method takes care of the temporal dependence (clusters of volatility) in the data. We follow Hall et al. (1995) and set the optimal block length equal to $T^{1/3}$.

[13] Starting from the asymptotic normality and the observed bias-variance trade-off for the Hill estimator, Goldie and Smith (1987) proposed to select the number of order statistics optimally by minimizing the Asymptotic Mean Squared error (AMSE). It follows that a stable region should exist in a plot of the estimator as function of the highest order statistics. The expectational linkage estimator exhibits the same bias-variance trade off and threshold selection can therefore proceed in the same graphical way.

Table 1
Extreme bilateral currency linkages.

	Panel A: Western currency pairs				
	DEM	CHF	JPY	GBP	AUD
DEM	–				
CHF	1.590 (200)	–			
JPY	1.105 (200)	1.120 (200)	–		
GBP	1.120 (200)	1.080 (100)	1.03 (200)	–	
AUD	1.115 (200)	1.120 (300)	1.09 (300)	1.132 (400)	–
	Panel B: Asian currency pairs				
	THB	IDR	PHP	HKD	MYR
THB	–				
IDR	1.060 (100)	–			
PHP	1.190 (100)	1.090 (200)	–		
HKD	1.002 (600)	1.002 (600)	1.002 (600)	–	
MYR	1.170 (600)	1.160 (200)	1.185 (200)	1.005 (200)	–
	Panel C: Latin American currency pairs				
	MXN	CLP	BOB	COP	VEB
MXN	–				
CLP	1.060 (100)	–			
BOB	1.005 (400)	1.000 (200)	–		
COP	1.060 (200)	1.035 (200)	1.010 (200)	–	
VEB	1.000 (200)	1.010 (200)	1.002 (400)	1.010 (200)	–

5.2. Empirical results

In this section, we measure the systemic instability of global currency markets by means of Eq. (18). We distinguish between industrial and emerging currency blocks. All currencies are expressed against the US dollar as the numeraire currency. Data were obtained from Datastream inc. We downloaded daily nominal bilateral exchange rates against the Pound sterling (GBP) as this renders the largest rectangular panel data set (largest cross section of currencies over the longest possible time span). Next, cross rates against the US dollar (USD) were calculated by applying the no triangular arbitrage condition. Hence, the USD is taken as the numeraire. For comparability we build three currency blocks of equal size (five currencies): Major industrial currencies, referred to as Western currencies, East Asian currencies and Latin American currencies. The industrial block covers the Deutsche Mark (DEM, and since January 1999 the euro), the Japanese Yen (JPY), the Pound sterling (GBP), the Swiss franc (CHF) and the Australian dollar (AUD). The Asian block includes the Hong Kong dollar (HKD), the Indonesian rupiah (IRD), the Malaysian ringgit (MYR), the Philippine peso (PHP), and the Thai baht (THB). Finally, as Latin American currencies were selected the Bolivian boliviano (BOB), Chilean peso (CLP), Colombian peso (COP), Mexican peso (MXN) and Venezuelan bolivar (VEN). Forex returns are calculated as log first differences, with the United States dollar serving as the common base currency. Our sample ranges from January 3, 1994 until September 13, 2006 which amounts to $T=3313$ daily prices.

We first calculated the linkage measure for currency pairs within each of the currency blocks.[14] The bivariate estimation results are contained in Table 1.

The value of $M_{\max}$ is reported between brackets. Each estimate of E has a direct economic interpretation. For example, a conditional expectation estimate of 1.590 for the ($US\$/CHF, US\$/DEM$) means that these exchange rates are expected to jointly crash more than half (59%) of the times if at least one of the two sharply drops in value. Striking is the heterogeneity in outcomes, both within and across currency blocks. An explanation for this heterogeneity is the difference in magnitudes of the currency fundamentals' scale parameters a and b_i, see the discussion after Corollary 1. The degree of similarity in fundamental volatilities within a currency block relative to the benchmark currency will determine the potential for systemic currency crises. Upon comparing the bivariate outcomes across currency blocks, one observes that bilateral crisis linkages versus the US dollar base currency are on average somewhat larger for Western currencies than for the Asian and Latin American currency pairs. The dollar therefore appears to be more important in inducing large systemic fluctuations among industrialized currencies as compared to emerging currencies. Note that this does not say that the dollar was not pivotal in many Asian and Latin American countries. Many of those crises occurred when those countries borrowed in dollars. It only says that the dollar was not central to multiple currencies crashing together in Latin America (e.g. Brazil and Argentina displayed opposite behavior when Argentina had to give up its dollar currency board).

For the sake of comparison we calculate our linkage measure for the Western, Asian and Latin American currency blocks in their entirety. This implies that the dependency measure for an entire block is bounded between [1, 5] instead of [1, 2] for the bivariate

[14] For sake of space considerations and because there is already overwhelming evidence from previous studies, see e.g. Boothe and Glassmann (1987), Koedijk et al. (1990), Hols and de Vries (1991) or Huisman et al. (2002), we do not report univariate estimates of tail indices and marginal exceedance probabilities but the results are available from the authors upon request.

Table 2
Extreme multilateral currency linkages: estimates and equality tests.

	Full sample ($T=3,313$)	Jan. 97–Dec. 1999 ($T=783$)
Panel A: multilateral E-estimates		
West	1.280 (100)	1.16 (50)
AS	1.240 (100)	1.220 (50)
LA	1.005 (200)	1.100 (50)
Panel B: equality tests		
West = LA	9.76	0.771
West = AS	0.447	−0.53
AS = LA	3.31	1.106

case. We also assess whether the cross-continent differences in E are statistically significant by using the equality test Eq. (19). Estimates and test results are reported in Table 2.

With an eye towards some sensitivity analysis, estimation and testing is also performed for a subsample that encompasses the Asian crisis. The point estimates seem to confirm the bivariate results as the propensity towards joint currency crashes is highest for industrial currencies and lowest in Latin America; whereas South East Asia seems to take an intermediate position. However, the equality tests in panel B only point to a statistically significant difference between Asia and the developed world over the full sample. Finally, the Asian and Latin American linkage estimates somewhat rise over the subsample period but the cross sectional differences are all statistically insignificant.

The results do in a way reveal the much debated decoupling. Asian currencies and Latin American currencies in particular appear to be decoupled from the dollar, whereas the Western currencies are not. Note that these results were obtained on data prior to the credit crisis. The larger magnitudes of (some of) the linkage estimates for the Western currencies relative to emerging market currencies perhaps reflect the greater convergence in the fundamentals due to monetary and economic integration, to the extent that $a \sim b_i$ in Corollary 1. For Latin American currency linkages versus the US dollar, it may be the relatively larger scale of the emerging market fundamentals relative to the US dollar fundamentals that explain the low values of $E\{\kappa|\kappa \geq 1\}$. In terms of Corollary 1, this would mean that $a < b_i$.

We gathered some additional evidence for Corrollary 1 by estimating emerging market linkages with respect to emerging market numeraires. Corollary 1 predicts that emerging linkage estimates should increase because the emerging base currency fundamental

Table 3
Extreme bilateral currency linkages for emerging cross rates

	Panel A: Asian cross rate pairs			
	A.1: numeraire = Thai baht (THB)			
	IDR	PHP	HKD	MYR
IDR	–			
PHP	1.11 (200)	–		
HKD	1.095 (200)	1.320 (200)	–	
MYR	1.170 (200)	1.225 (200)	1.510 (200)	–
	A.2: numeraire = Indonesian rupiah (IDR)			
	THB	PHP	HKD	MYR
THB	–			
PHP	1.595 (200)	–		
HKD	1.660 (200)	1.710 (200)	–	
MYR	1.655 (200)	1.695 (200)	1.835 (200)	–
	Panel B: Latin American cross rate pairs			
	B.1: numeraire = Mex.peso (MXN)			
	CLP	BOB	COP	VEB
CLP	–			
BOB	1.342 (500)	–		
COP	1.252 (400)	1.347 (300)	–	
VEB	1.140 (200)	1.165 (200)	1.135 (200)	–
	B.2: numeraire = Venezolan bolivar (VEB)			
	MXN	CLP	BOB	COP
MXN	–			
CLP	1.580 (200)	–		
BOB	1.615 (200)	1.740 (100)	–	
COP	1.585 (200)	1.680 (200)	1.780 (100)	–

strongly resembles the other emerging market fundamentals ($a \sim b_i$), or because perhaps in some cases the numeraire fundamental even dominates the other fundamentals, i.e. $a > b_i$. Table 3 reports the linkage measure estimates for pairs of Asian and Latin American cross rates, see also the cross-plots in the Appendix. Two different domestic emerging market numeraires for both Asia and Latin America are considered per block. We observe that selecting the numeraire currencies inside the considered currency block usually pushes the linkage estimates upward, confirming the theory behind Corollary 1. Thus while we find evidence for decoupling of Latin American countries, these countries at the same time exhibit a strong interdependency among themselves, as do some of the Asian countries. The decoupling was what one saw during the first stages of the credit crisis. But as the crisis wears on, non-Western countries are likely going to be hit by the world wide slump. This may still trigger a currency crisis in and between these countries.

6. Conclusion

It is by now well known that financial returns exhibit heavy tails and thus are non-normally distributed. This implies that extreme market conditions tend to occur more frequently than expected on the basis of the normal distribution, which is used so often in standard asset pricing approaches. From the point of view of international financial stability and portfolio diversification, the strength of asset linkages during crisis periods matters even more, as these determine the stability of the system as a whole. This paper has shown how Mandelbrot's innovative work on the univariate properties of the speculative price series inspired us to investigate the multivariate properties of currency systems and their stability.

We show that in affine models of the (international) financial system the fragility of the system or its systemic stability hinges critically on the type of marginal distribution that applies to the country fundamentals. More specifically, we demonstrate that in linear models of forex returns, the nature of interdependency between different exchange rate returns in times of crisis is fundamentally related to the univariate properties of the distribution of the risk factors. Suppose that the logarithmic exchange rate returns are a linear function of the domestic and foreign fundamentals. This implies that different exchange rate returns against the same base currency are correlated because they share common fundamentals. Nevertheless, if one currency return crashes, the probability that the other currency breaks down as well vanishes asymptotically if the forex fundamentals exhibit thin tails, as is the case for the normal distribution. Alternatively, if the marginal distributions exhibit heavier tails than the normal, e.g. are Student-*t* distributed, the probability that the other currency breaks down as well remains strictly positive even in the limit. We therefore speak of, respectively, weak and strong crisis linkages between different currencies. Correspondingly, the international monetary and financial system may be characterized as relatively robust in the former case, where destabilizing phenomena like contagion do not occur systematically, while it is relatively fragile in the latter case. Our theoretical derivation has direct relevance for economic policy. By pursuing policies with a 'steady hand' instead of orchestrating drastic changes in variables like money supply, interest rates or public expenditure, public authorities can diminish the scope for fat tails in the fundamentals. In specific circumstances of large market-driven fluctuations, strong counteracting measures may be advisable. In the light of our argument, policy institutions may in this way contribute to the stability of the international exchange rate system.

Empirically we uncovered important differences between Western and non-Western currencies. The former reveal a higher joint connection to the dollar than the other currencies. This may be considered evidence for decoupling. However, this being said, we also find evidence for a stronger interdependency among the non-Western currency blocs than for the Western currencies.

Appendix

In this appendix we analyze the CLP/MXN versus the COP/MXN foreign exchange rate returns using crossplots and the linkage estimator. We start with three crossplots. A crossplot is already quite revealing for the dependency in the data. The first crossplot is for the daily exchange rate returns of the CLP/MXN versus the COP/MXN. Note that many of the extreme realizations lie on the diagonal, pointing to strong dependency in the tail areas.

The next two plots present remakes of this crossplot for simulated data under the assumption of correlated normals and correlated Student-*t* (3 degrees of freedom) with correlation coefficient 0.819, as in the forex data and with the same standard deviations as well. The Student-*t* plot also has several outliers along the diagonal, while the normal crossplot is still a nice ellipse with few outliers (Figs. 1 and 2).

The next three plots give the behavior of the linkage estimator for the currency data (Fig. 3)

$$E = \frac{P\{CLP / MXN > s\} + P\{COP / MXN > s\}}{1 - P\{CLP / MXN \le s, COP / MXN \le s\}}$$

and for the simulated data. In the plots the threshold s is varied from very high on the left hand side to very low (inside the sample on the right hand side in the plot). Fig. 4 shows the plot for the exchange rate data. Close to the origin, there are so few excesses that the plot is rather unstable, while it settles at a stable plateau after using the first hundred top observations.

Next are the linkage measure plots for the simulated data. The plot for the Student-*t* data is similar to the plot for the forex data. The normal plot is quite different as the plot first lingers in the neighborhood of one due to the asymptotic independence. The Student-*t* data generate a plot which immediately jumps up. For the Student-*t* data the limiting theoretical value for the linkage measure, given the correlation of 0.81 and standard deviations is 1.51. The latter value implies that one out of two crashes is a joint crash (Figs. 5 and 6).

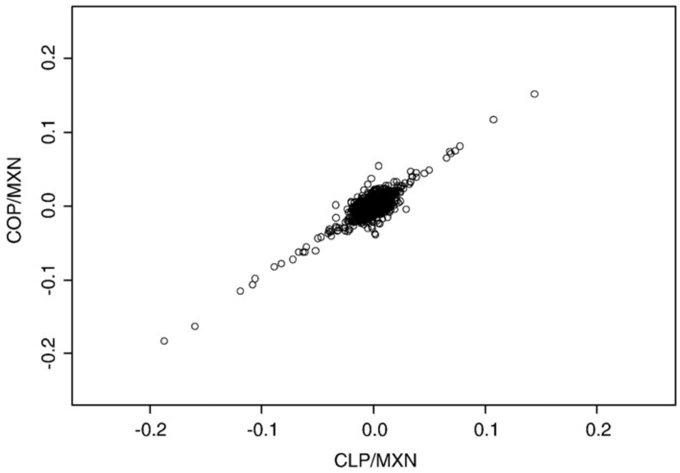

Fig. 1. CLP/MXN and COP/MXN.

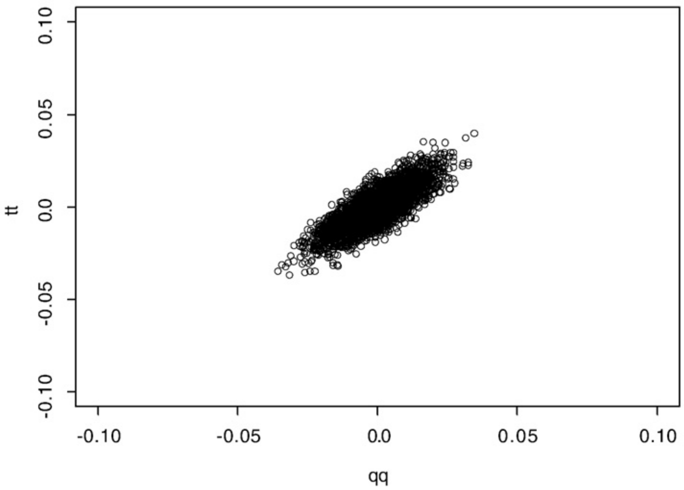

Fig. 2. Crossplot correlated normals, rho = 0.819.

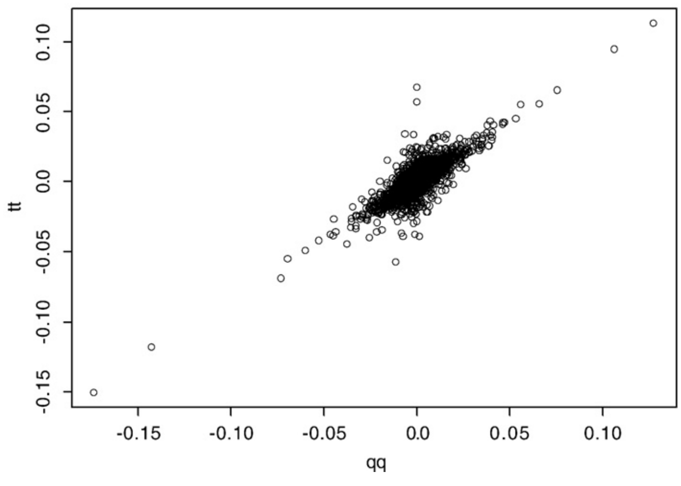

Fig. 3. Crossplot correlated Student-*t* 3df, rho = 0.819.

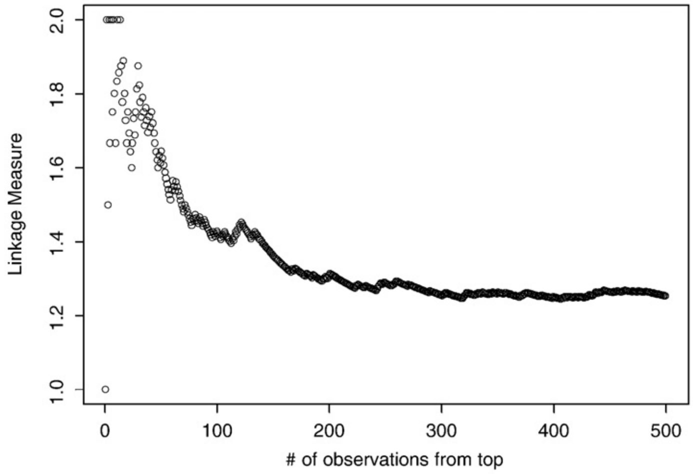

Fig. 4. CLP/MXN and COP/MXN.

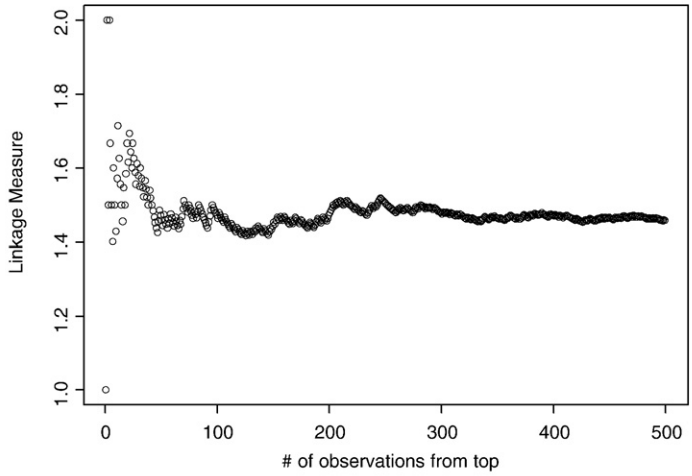

Fig. 5. Correlated Student-*t* 3df, rho = 0.819.

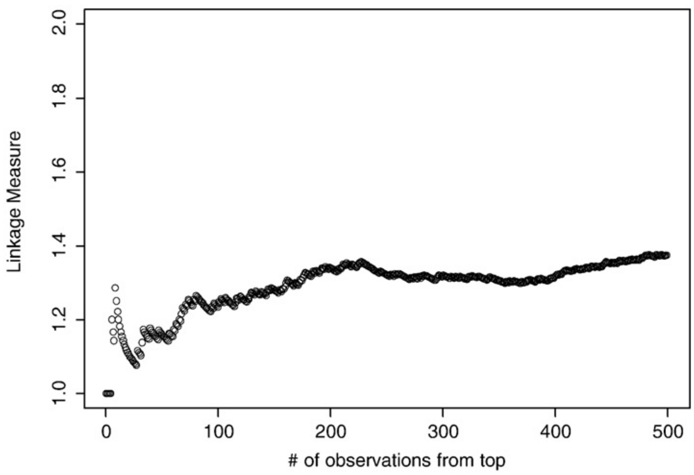

Fig. 6. Correlated normals, rho = 0.819.

References

Aghion, P., Bachetta, P., Banerjee, A., 2001. Currency crises and monetary policy in an economy with credit constraints. European Economic Review 54, 1121–1150.

Ang, A., Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Economics 63 (3), 443–494.

Balkema, A.A., de Haan, L., 1974. Residual life time at great age. Annals of Probability 2, 792–804.

Boothe, P., Glassmann, D., 1987. The statistical distribution of exchange rates: empirical evidence and economic implications. Journal of International Economics 22, 297–320.

Chapernowne, D.G., 1953. A model of income distribution. The Economic Journal 318–351.

Danielsson, J., de Haan, L., Peng, L., de Vries, C.G., 2001. Using a bootstrap method to choose the sample fraction in tail index estimation. Journal of Multivariate Analysis 76, 226–248.

De Bandt, O., P. Hartmann (2000), 'Systemic risk: a survey', ECB Working Paper, no. 35, November.

De Haan, L., Resnick, S., Rootzen, H., de Vries, C.G., 1989. Extremal behavior of solutions to a stochastic difference equation, with applications to ARCH processes. Stochastic Processes and their Applications 32, 213–224.

De Haan, L., Jansen, D.W., Koedijk, K., de Vries, C.G., 1994. Safety first portfolio selection, extreme value theory and long run asset risks, in J. Galambos ed., Proceedings from a Conference on Extreme Value Theory and Applications, Kluwer Press, 471–487.

De Vries, C.G., 2005. The simple economics of bank fragility. Journal of Banking and Finance 29, 803–825.

Diamond, D., Dybvig, P., 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 91 (3), 401–419.

Embrechts, P., McNeil, A., Straumann, D., 1999. Correlation: pitfalls and alternatives. Risk 12 (5), 69–71.

Engle, R., 1982. Autoregressive conditional heteroscedastic models with estimates of the variance of inflation in the United Kingdom inflation. Econometrica 50, 987–1007.

Feller, W., 1971. 2nd ed. An Introduction to Probability Theory and its Applications, vol. II. Wiley, New York.

Forbes, K.J., Chinn, M.D., 2003. A decomposition of global linkages in financial markets over time. NBER Working Paper, no. 9555.

Frenkel, J.A., 1976. A monetary approach to the exchange rate: doctrinal aspects and empirical evidence. Scandinavian Journal of Economics 200–224.

Gabaix, X., Parameswaran, G., Vasiliki, P., Stanley, H.E., 2003. A theory of power law distributions in financial market fluctuations. Nature 423, 267–270.

Gencay, R., Selcuk, F., 2006. Overnight borrrowing, interest rates and extreme value theory. European Economic Review 50, 547–563.

Goldie, G.M., Smith, R., 1987. Slow variation with remainder: theory and applications. Quarterly Journal of Mathematics 38, 45–71.

Gorton, G., 1988. Banking panics and business cycles. Oxford Economic Papers 40, 751–781.

Hall, P., Horowitz, J., Jing, B., 1995. On blocking rules for the bootstrap with dependent data. Biometrika 82 (3), 561–574.

Hartmann, P., Straetmans, S., de Vries, C.G., 2003. A global perspective on extreme currency linkages'. In: Hunter, W.C., Kaufman, G.G., Pomerleano, M. (Eds.), Asset Price Bubbles: The Implications for Monetary. Regulatory and International Policies. MIT Press, Cambridge (MA), pp. 361–382.

Hartmann, P., Straetmans, S., de Vries, C.G., 2004. Asset market linkages in crisis periods. Review of Economics and Statistics 81, 19–24.

Hols, M.C.A.B., de Vries, C.G., 1991. The limiting distribution of extremal exchange rate returns. Journal of Applied Econometrics 6, 287–302.

Huang, X. (1992), Statistics of Bivariate Extreme Values, PhD thesis, Tinbergen Institute.

Huisman, R., Koedijk, K.G., Kool, C., Palm, F., 2002. The tail fatness of FX returns reconsidered. De Economist 150, 401–403.

Koedijk, K.G., Schafgans, M.M.A., de Vries, C.G., 1990. The tail index of exchange rate returns. Journal of International Economics 29, 93–108.

Krugman, P., 1979. A model of balance of payments crises. Journal of Money, Credit, and Banking 11, 311–325.

Mandelbrot, B., 1963a. The variation of certain speculative prices. Journal of Business 36, 394–419.

Mandelbrot, B., 1963b. New methods in statistical economics. Journal of Political Economy 61, 421–440.

Masson, P., 1999. Contagion: macroeconomic models with multiple equilibria. Journal of International Money and Finance 18, 587–602.

Obstfeld, M., 1986. Rational and self-fulfilling balance of payments crises. American Economic Review 76, 72–81.

Poon, S.H., Rockinger, M., Tawn, J., 2004. Extreme-value dependence measures in financial markets: Diagnostics, models and financial implications. The Review of Financial Studies 17, 581–610.

Ross, S.A., 1976. The arbitrage theory of capital asset pricing. Journal of Economic Theory 13, 341–360.

Ruben, H., 1964. An asymptotic expansion for the multivariate normal distribution and Mill's ratio, *Journal of Research of the National Bureau of Standards-B*. Mathematics and Mathematical Physiscs 68B (1), 3–11.

Sibuya, M., 1960. Bivariate extreme statistics. Annals of the Institute of Statistical Mathematics 11, 195–210.

Slijkerman, J.F. (2007), Financial Stability in the EU: Quantifying the downside risk of financial institutions, PhD thesis, Tinbergen Institute.

Starica, C., 1999. Multivariate extremes for models with constant conditional correlations. Journal of Empirical Finance 6, 515–553.