

## EXTREME US STOCK MARKET FLUCTUATIONS IN THE WAKE OF 9/11

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### SUMMARY

We apply extreme value analysis to US sectoral stock indices in order to assess whether tail risk measures like value-at-risk and extremal linkages were significantly altered by 9/11. We test whether semi-parametric quantile estimates of ‘downside risk’ and ‘upward potential’ have increased after 9/11. The same methodology allows one to estimate probabilities of joint booms and busts for pairs of sectoral indices or for a sectoral index and a market portfolio. The latter probabilities measure the sectoral response to macro shocks during periods of financial stress (so-called ‘tail- $\beta$ s’). Taking 9/11 as the sample midpoint we find that tail- $\beta$ s often increase in a statistically and economically significant way. This might be due to perceived risk of new terrorist attacks. Copyright © 2008 John Wiley & Sons, Ltd.

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### 1. INTRODUCTION

Does US common stock exhibit a higher propensity toward sharp price declines since the dreadful 9/11 events? Do sharp drops in stock prices tend to co-move more frequently since 9/11? Most financial practitioners would probably give a positive answer to both questions. Answering these two questions is crucial from a regulatory (potential 9/11 impact on US systemic stability) and risk management point of view (potential 9/11 impact on the scope for risk diversification during times of market stress). The more stocks or sectoral indices jointly drop in value, the more in danger are even large investment banks and institutional investors that hold widely diversified trading portfolios. The number of stocks or sectors affected by a crisis situation may also determine the severity of any real effects that might follow.

The question arises why one would expect a lasting impact of 9/11 in the financial markets. Empirical evidence suggests that US common equity rapidly recovered in the aftermath of 9/11 (see, for example, Chen and Siems, 2004). However, 9/11, the Madrid and London bombings, as well as the Al-Qaeda threats toward the US-led ‘War on Terror’ coalition created the perception of a globalization of ‘terrorism risk’ (see de Mey, 2003; Brown *et al.*, 2004). This may well have increased systematic risk in the equity markets. A number of event studies investigated the 9/11 impact on a few sectors like airlines (Drakos, 2004) and the (re)insurance business (Kunreuther and Michel-Kerjan, 2004).<sup>1</sup>

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<sup>1</sup> In the aftermath of 9/11 the insurance business terminated coverage of terrorist damage in order to limit the systemic risk for the insurance industry. The November 2002 Terrorism Risk Insurance Act (TRIA) partly solved this ‘market

This paper extends the scant 9/11 finance literature with a volatility and dependence analysis of extreme events for different indices of US common stock on a sectoral level. More specifically, we try to assess whether 9/11 has a statistically and economically significant impact on our volatility and co-movement measures. We opt for a sectoral focus because some sectors are by nature more vulnerable to terrorist attacks than others (e.g., banking, insurance, transportation or public utilities). The study of asset return linkages during crisis periods is not new, although most previous studies focused on cross-country linkages between asset returns. The bulk of the earlier contributions implement some type of correlation analysis, often based on multivariate GARCH or stochastic volatility models. These articles typically study whether financial markets are more strongly co-moving during periods of market stress compared to periods of market quiescence and also question the direction of international spillovers (see King and Wadwhani, 1990; Lin *et al.*, 1994; Susmel and Engle, 1994). An increasingly important subset of this ‘market linkages’ literature focuses on whether financial crises are ‘contagious’ (see Forbes and Rigobon, 2002; Bae *et al.*, 2003; Chan-Lau *et al.*, 2004). Hartmann *et al.* (2005) argue that the contagion concept is far from unambiguously defined and classify the most frequent interpretations in the literature.

The main objection against the (bulk of the) market linkages literature is that it is so highly correlation oriented. However, correlations can be very misleading indicators of dependence during crisis episodes. First, correlations are nonrobust to changing the underlying distributional assumptions of the return processes. For example, Ang and Chen (2002) demonstrate for the bivariate normal distribution that the correlation varies considerably when truncated (i.e., defined over a subset of returns) and eventually goes to zero in the case of two-variable truncation in the bivariate tail. In addition, the truncated correlation differs across different classes of multivariate distributions; also, correlations can only capture linear dependence, whereas one might suspect crisis spillovers to be fundamentally nonlinear phenomena. For a more in-depth treatment of the pitfalls of correlation analysis, see, for example, Embrechts *et al.* (1999).

Mainly because of these concerns regarding the applicability of covariance analysis during periods of high market volatility, a growing body of literature applies extreme value analysis (EVT). Loosely speaking, EVT enables one to estimate marginal and joint probabilities of infrequent tail events like crises without the need to resort to a parametric probability law for the returns. As will be discussed in the estimation section of this paper, some mild conditions on the tail behavior of the returns suffice for the purpose of estimation and statistical inference.

Moreover, EVT allows one to focus on crisis phenomena that are more severe and abrupt than the ones mainly captured by more standard econometric techniques. This ensures that what we will estimate truly reflects sectoral stock linkages in stress periods. Finally, one has to be aware that the EVT approach focuses on the unconditional distribution of returns in contrast to stochastic volatility type of models that produce time-varying measures of volatility and dependence. Conditional models will be preferred by risk managers and investors with short time horizons for the sake of short-term volatility forecasting. However, in this paper, we focus on measures of sectoral system stability which might be used as building blocks for regulatory frameworks. To assess stability of sectors (or the whole financial system) supervisors like to know how likely it is that one or several sectors collapse given that other sectors break down, or how likely it is that one or several sectors collapse given that there is an adverse aggregate shock. However, regulations to prevent these types of systemic domino effects are not determined or changed

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incompleteness’ by letting the government play the role of ‘insurer of last resort’ in case of massive terrorist damage. However, TRIA does not cover nuclear, chemical, and biological hazards.

overnight. They are preferably based on long-term unconditional risk measures instead of short-term volatility predictions that exploit volatility persistence. This is why for the questions we are focusing upon straight return spillovers are preferable to volatility spillovers and unconditional modeling is preferable to conditional models.<sup>2</sup>

This paper's contribution to the literature on the impact of 9/11 is twofold. First, we apply the novel multivariate EVT techniques proposed by Ledford and Tawn (1996) and by Poon *et al.* (2004) to estimate the level of 'sectoral' risk before and after 9/11. We distinguish univariate measures of tail risk (tail quantiles or 'value-at-risk' levels) from bivariate measures of systematic tail risk (co-exceedance probabilities defined on the bivariate tail of the joint return distribution). Co-exceedance probabilities for pairs of sectoral returns reflect the potential for sectoral problems to spill over from one sector to another. As such it can be interpreted as a measure for contagion risk. In addition, one can also calculate the co-exceedance probability of sectoral indices together with variables that are supposed to be transmitters of macro shocks (market indices, yield spreads, oil prices etc.). This second type of co-exceedance probability is interpretable as the tail equivalent to standard asset pricing measures of systematic risk like the CAPM- $\beta$ ; we will therefore also call it a 'tail- $\beta$ '. The second contribution of the paper consists in assessing whether tail quantiles and co-exceedance probabilities are stable across upper and lower tails (asymmetry hypothesis) and across time (structural change hypothesis). As to date, these types of tests have hardly been considered within an EVT framework. Asymmetry tests for co-exceedance probabilities extend an existing literature on (linear) tail correlation asymmetry (see, for example, Longin and Solnik, 2001; Ang and Chen, 2002) into a more general (possibly nonlinear) tail dependence framework. Testing for structural change in the tail behavior of the unconditional distribution is important both from a purely statistical and from a policy perspective. The statistical implication of structural change is that the application of EVT over long time spans becomes problematic when tail properties of the unconditional distribution are nonconstant. From a policy perspective, structural breaks in the form of an increase in the co-exceedance probability can be interpreted as a rise in systemic risk or a decreased potential for diversifying tail risk.<sup>3</sup>

The paper is organized as follows. The next section introduces the co-exceedance probability measure as a device for extremal dependence measurement; we also discuss EVT procedures for estimation and statistical inference (asymmetry and structural change tests). Section 3 contains the empirical results. We distinguish between univariate estimation results (tail indices and extreme quantiles for univariate sectoral tails) and bivariate estimation results (tail- $\beta$ s and sectoral co-exceedance probabilities). Estimation results are complemented with structural change and asymmetry testing results. Conclusions are drawn in Section 4.

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<sup>2</sup> In univariate and bivariate settings EVT has been previously implemented to assess the severity of extreme market (co-)movements. For example, Koedijk *et al.* (1990) study the (heavy) tails of foreign exchange rate returns. Jansen and de Vries (1991) and Longin (1996) analyze stock market booms and busts whereas de Haan *et al.* (1994) consider extreme upturns and downturns in bond markets. Bivariate EVT has been employed to measure extreme stock market spillovers in either a parametric (Longin and Solnik, 2001) or semi-parametric fashion (see Straetmans, 2000; Poon *et al.*, 2004). Hartmann *et al.* (2003, 2004) address various forms of currency and stock-bond spillovers.

<sup>3</sup> Studies on structural breaks in the tail index  $\alpha$  include, for example, Koedijk *et al.* (1990) and Jansen and de Vries (1991) for exchange rates and stock markets, respectively. Tail index asymmetry has been investigated by, for example, Jansen and de Vries (1991) and Jondeau and Rockinger (2003). Longin and Solnik (2001) test for asymmetric tail correlations in the international equity market using a bivariate logistic model for the tail copula. Hartmann *et al.* (2004) opted for a semi-parametric tail copula approach in order to test for asymmetries in stock-bond markets spillovers.

## 2. A TAIL EQUIVALENT FOR BETA

This section starts with a formal definition of the co-exceedance probability measure and a discussion of some potential applications. Next, we introduce semi-parametric estimation procedures for the co-exceedance probability and the univariate extreme quantile. We end the section by formulating test statistics for the null hypotheses of no structural change and absence of asymmetry across the return tails.

### 2.1. Theory

Suppose one is interested in measuring the probability of a stock price collapse conditional on a stock price collapse for another company or sectoral index. This probability reflects the dependence between the two stock returns during times of market stress. Let the two return series be represented by random variables  $X_1$  and  $X_2$ . We adopt the convention to take the negative of stock returns, so that all expressions are defined on the upper return tails. Without loss of generality we choose the tail quantiles  $Q_1$  and  $Q_2$  such that the tail probabilities are the same across stocks, i.e.,  $P\{X_1 > Q_1(p)\} = P\{X_2 > Q_2(p)\} = p$ . Despite a common exceedance probability (or ‘marginal significance level’)  $p$ , the quantiles  $Q_1$  and  $Q_2$  will generally differ because the marginal distribution functions for  $X_1$  and  $X_2$  are company specific (in the case of individual stocks) or portfolio specific (in the case of portfolios). A common  $p$  makes the corresponding tail quantiles or extreme ‘value-at-risk’ levels  $Q_1$  and  $Q_2$  better comparable across assets or portfolios.

From elementary probability theory (starting from the standard definition of conditional probability) we can now easily write down a bivariate probability measure by using the notation introduced above:

$$\begin{aligned}\tau_\beta &\equiv P\{X_1 > Q_1(p) | X_2 > Q_2(p)\} \\ &= \frac{P\{X_1 > Q_1(p), X_2 > Q_2(p)\}}{P\{X_2 > Q_2(p)\}} \\ &= \frac{P\{X_1 > Q_1(p), X_2 > Q_2(p)\}}{p}\end{aligned}\tag{1}$$

Conditional exceedance probabilities for higher dimensions than two can be straightforwardly defined in the same manner (see, for example, Hartmann *et al.*, 2005). The probability measure  $\tau_\beta$  reflects the strength of the interdependence for the return pair  $(X_1, X_2)$  beyond thresholds  $Q_1$  and  $Q_2$ . Notice that  $\tau_\beta$  reduces to  $p^2/p = p$  under complete independence.

If the conditioning asset  $X_2$  is a ‘market’ portfolio like, for example, NYSE Composite or NASDAQ Composite, the co-exceedance probability can be interpreted as a natural (tail) extension of the regression-based CAPM- $\beta$ . We will therefore call it a tail- $\beta$  in these circumstances. Tail- $\beta$ s will be reported with respect to the NYSE Composite market index and an oil index. In addition, the conditional probability (1) will also be calculated for pairs of sectoral stock index portfolios in order to assess the potential for extreme sectoral spillovers. The latter probabilities can be interpreted as reflecting extreme sectoral ‘contagion’ risk (see Chan-Lau *et al.*, 2004). Whereas the contagion and tail- $\beta$  interpretations of (1) might appeal to financial regulators, risk managers can use the co-exceedance probability as a device for stress testing risky positions (one could think of  $X_1$  as representing the return on a portfolio or risky trading position on a corporation’s

balance sheet). The conditioning event  $|X_2 > Q_2$  may reflect any type of stress scenario like a sharp drop in Asian markets, interest rates, yield spreads, etc. A more detailed exposition on how to use (1) as a stress-testing device will be provided at the end of the next subsection.

## 2.2. Estimation

Our empirical investigation consists of a univariate and bivariate extreme value analysis of the tail behavior for US sectoral index returns. We first estimate extreme tail quantiles  $Q(p)$  in the univariate part. In the bivariate part we report estimates of the co-exceedance probability (1).

Univariate EVT builds on the well-known generalized extreme value (GEV) distribution, which is the limit law for (appropriately scaled) maxima of a stationary process. Broadly speaking, there are two families of univariate EVT techniques that differ in the way the parameters of the GEV distribution are estimated. In a first approach, one fits block maxima to the GEV distribution by means of maximum likelihood optimization. The maxima approximately follow the GEV distribution provided the blocks are sufficiently long (e.g., yearly). Peaks-over-threshold (POT) models constitute a second set of techniques. Parametric POT models hinge upon maximum likelihood optimization and exploit the property that the distribution of excess losses over a given high threshold converges to a generalized Pareto distribution (GPD); but one can also fit the distributional tail beyond some high threshold in a semi-parametric way.<sup>4</sup> We opted for the latter approach.

We start from the stylized fact that financial returns exhibit ‘heavy’ tails. Loosely speaking, this implies that the marginal exceedance probability for a return series  $X$  as a function of the corresponding quantile can be approximately described by a power law (or regularly varying tail):

$$P\{X > x\} \approx l(x)x^{-\alpha}, \quad x \text{ large} \quad (2)$$

and where  $l(x)$  is a slowly-varying function (i.e.,  $\lim_{x \rightarrow \infty} l(tx)/l(x) = 1$ , for all fixed  $t > 0$ ).

The parameter  $\alpha$  is called the tail index and determines the tail probability’s rate of decay if  $x$  is increased. Clearly, the lower  $\alpha$  the slower the probability decay and the higher the probability mass in the tail of  $X$ . The regular variation property implies that all distributional moments higher than  $\alpha$ , i.e.,  $E[X^r]$ ,  $r > \alpha$ , are unbounded, signifying the ‘fat tail property’. Popular distributional models like the Student- $t$ , symmetric stable or the generalized autoregressive conditional heteroscedasticity (GARCH) model with conditionally normal errors all exhibit this tail behavior.

Univariate extreme quantiles for  $X$  can now be estimated by using the semi-parametric quantile estimator from de Haan *et al.* (1994):

$$\hat{q}_p = X_{n-m,n} \left( \frac{m}{pn} \right)^{\frac{1}{\alpha}} \quad (3)$$

<sup>4</sup>Examples of parametric GEV and GPD fitting can be found in Longin (1996), Nefci (2000) and Bali and Nefci (2003). Semi-parametric tail estimation approaches include Dekkers and de Haan (1989), Jansen and de Vries (1991) and Danielsson and de Vries (1997). Longin (1996) and Bali (2003) also consider a regression-based approach in order to determine the parameters of GEV or GPD. Their nonlinear regressions fit the relative frequencies (empirical probabilities) of the historical return data to the corresponding cumulative probabilities implied by the GPD and GEV probability models.

and where the ‘tail cut-off point’  $X_{n-m,n}$  is the  $(n-m)$ th ascending order statistic (or loosely speaking the  $m$ th smallest return) from a sample of size  $n$  such that  $q > X_{n-m,n}$ . An important aspect of the estimator  $\hat{q}_p$  is that it can extend the empirical distribution function outside the domain of the sample by means of its asymptotic Pareto tail from (2).<sup>5</sup> The estimator (3) is still conditional upon knowing the tail index  $\alpha$ . We estimate the tail index by means of the popular Hill (1975) estimator:

$$\hat{\alpha} = \left( \frac{1}{m} \sum_{j=0}^{m-1} \ln \left( \frac{X_{n-j,n}}{X_{n-m,n}} \right) \right)^{-1} \quad (4)$$

where  $m$  has the same value and interpretation as in (3). Further details on the Hill estimator and related procedures to estimate the tail index are provided in Jansen and De Vries (1991) or the monograph by Embrechts *et al.* (1997).<sup>6</sup>

The Hill statistic (4) still requires a choice of the number of highest-order statistics  $m$  used in estimation. Goldie and Smith (1987) suggest selecting  $m$  such as to minimize the asymptotic mean-squared error (AMSE) of the Hill statistic. This minimum should exist because of the bias–variance trade-off that is characteristic of the Hill estimator. Balancing the bias and variance constitutes the starting point for most empirical techniques to determine  $m$ . We opted for the Beirlant *et al.* (1999) algorithm, which exploits an exponential regression model (ERM) on the basis of scaled log-spacings between subsequent extreme order statistics from a Pareto-type distribution. Running least squares regressions on this exponential regression model allows one to estimate the empirical AMSE for different  $m$ -values and to choose the optimal  $m$  that minimizes the AMSE.<sup>7</sup>

In order to estimate the co-exceedance probability (1), it suffices to calculate the joint probability in the numerator of (1). Bivariate EVT theory basically offers two types of estimation approaches. A first approach hinges upon the so-called ‘stable tail dependence function’ (STDF) or ‘tail copula’ of  $(X_1, X_2)$  (see, for example, Embrechts *et al.*, 2000). The co-exceedance probability is related to the STDF via the following chain of equalities that follow from elementary probability calculus:

$$P\{X_1 > Q_1(p), X_2 > Q_2(p)\} = 2p - p_{12}$$

with  $p_{12} = P\{X_1 > Q_1(p) \text{ or } X_2 > Q_2(p)\}$ . The stable tail dependence function (STDF) can be used to approximate  $p_{12}$ . For sufficiently small  $t > 0$ , the STDF function  $l(u, v)$  exists such that

$$l(u, v) \approx t^{-1}P\{X_1 > Q_1(tu) \text{ or } X_2 > Q_2(tv)\}$$

for small but positive values  $u, v$ . Choose  $tu = tv = p$ , so that  $l(u, v) = l(t^{-1}p, t^{-1}p)$ . However, the linear homogeneity property of the STDF implies  $tl(t^{-1}p, t^{-1}p) = l(p, p)$ . Hence, for a

<sup>5</sup> The estimator (3) is a first-order Taylor approximation of the true tail quantile. How good this approximates the true tails has been previously studied by, for example, Danielsson and de Vries (1997). We performed our own simulation study for a variety of data-generating processes and found that the performance of the quantile estimator is quite satisfactory. The simulation study is available from the authors upon request.

<sup>6</sup> Pareto tail decline is one of three subclasses of limit laws nested into the GEV distribution (the other two are the fat-tailed Weibull df and the thin-tailed Gumbel df). We investigated the empirical validity of the heavy tail corroboration by estimating the tail shape parameter  $\gamma$  using the Dekkers *et al.* (1989) estimator. Whereas the Hill estimator is only valid for regularly varying tails, the DEDH estimator behaves well under all three limit laws. Thin-tailed returns correspond to  $\gamma = 0$  while Weibull limit behavior implies  $\gamma < 0$ . We found that  $\hat{\gamma} > 0$  for nearly all tails. Moreover, the positive sign is nearly always statistically significant. Calculations are available upon request.

<sup>7</sup> The optimal  $m$ -values are not included in tables or figures for space considerations but are available upon request.

marginal significance probability  $p$  that is sufficiently small, we obtain  $l(p, p) \approx p_{12}$ . The tail copula can be shown to be one-to-one with the bivariate extreme value distribution of the scaled maxima for  $(X_1, X_2)$ .<sup>8</sup> The curvature of  $\ell(\cdot, \cdot)$  completely determines the dependency structure between the  $(X_1, X_2)$  components in the tail area. A basic property of  $\ell(\cdot, \cdot)$  constitutes the inequality

$$\max(u, v) \leq \ell(u, v) \leq u + v \quad (5)$$

Equality holds on the left-hand side if the equity returns are completely mutually dependent in the tail area, while equality on the right-hand side is obtained when returns are mutually independent in the tail area ('tail' independence).<sup>9</sup> One may either estimate tail copula by means of maximum likelihood based on a parametric choice for the tail or by implementing semi-parametric estimation procedures. Longin and Solnik (2001) calculate tail correlations for equity markets using a bivariate logistic tail copula, whereas Hartmann *et al.* (2004) use a semi-parametric measure for  $\ell(\cdot, \cdot)$  in order to study bilateral crisis linkages between stock and bond markets.

The weakness of this approach is that it presupposes tail dependence. However, this property is not necessarily present in bivariate data.<sup>10</sup> As we do not want to impose the asymptotic dependence restriction, we opted for the more flexible EVT approach proposed by Ledford and Tawn (1996) (for another recent finance application see, for example, Poon *et al.*, 2004). In a nutshell, this technique consists in generalizing the (univariate) empirical stylized fact of 'fat-tailed' equity returns toward the bivariate tails on which the tail probability (1) is conditioned. Before proceeding with the modeling of the extreme dependence structure, however, it is worthwhile eliminating any possible influence of marginal aspects on the joint tail probabilities by transforming the original variables to a common marginal distribution. After such a transformation, differences in joint tail probabilities are solely attributable to differences in the tail dependence structure. Thus our dependence measures, unlike correlation, for example, are no longer influenced by the differences in marginal distributions. In this spirit we transform stock index returns  $(X_1, X_2)$  to unit Pareto marginals:

$$\tilde{X}_i = \frac{1}{1 - F_i(X_i)}, \quad i = 1, 2 \quad (6)$$

with  $F_i(\cdot)$  representing the marginal cumulative distribution function for  $X_i$ .<sup>11</sup> Any monotonically increasing variable transform like (6) leaves the co-exceedance probability (1) invariant which

<sup>8</sup> The tail copula function is interpretable as a tail version of the copula. The copula of a joint distribution  $F(\cdot, \cdot)$  can be represented by  $D(u, v) = F(F_1^{-1}(u), F_2^{-1}(v))$  for  $0 \leq u, v \leq 1$  and with  $F_i^{-1}(i = 1, 2)$  the generalized marginal inverses. In contrast to the original distribution function, the copula only reflects dependence information because the marginals have been transformed to uniform distributions. It easily follows that  $l(u, v) = \lim_{t \rightarrow 0} t^{-1} [1 - D(1 - tu, 1 - tv)]$  (see, for example, Embrechts *et al.*, 2000).

<sup>9</sup> Note that independence over the full range of the joint return distribution implies that  $F(x, y) = F_X(x)F_Y(y)$  irrespective of the quantile magnitudes  $(x, y)$ , whereas tail independence only requires this factorization to hold for large  $(x, y)$ . Thus non-extreme return pairs can be dependent even if the extremes are tail independent. The multivariate normal distribution with  $\rho \in (-1, 1)$  and  $\rho \neq 0$  constitutes an example.

<sup>10</sup> Semi-parametric estimation procedures for  $\ell(\cdot, \cdot)$  typically exploit linear homogeneity, i.e.,  $\ell(\lambda p_1, \lambda p_2) = \lambda \ell(p_1, p_2)$  with  $\lambda > 0$ . However, homogeneity breaks down in the case of tail independence and semi-parametric estimators for  $\ell(\cdot, \cdot)$  exhibit degenerate limiting distributions (see, for example, Hartmann *et al.*, 2004).

<sup>11</sup> Since  $F_i(i = 1, 2)$  are unknown, we replace them with their empirical counterparts.

$$\hat{F}_i(X_{ij}) = \frac{R_{X_{ij}}}{n+1}, \quad i = 1, 2; j = 1, \dots, n$$

implies

$$P\{X_1 > Q_1(p), X_2 > Q_2(p)\} = P\{\tilde{X}_1 > s, \tilde{X}_2 > s\}$$

with  $s = 1/p$ . Thus, one does not need to know the values of the univariate quantiles  $Q_1$  and  $Q_2$  in order to calculate the joint probability as they are mapped to the common quantile  $s$ . The estimation problem can be trivially reduced to estimating a univariate exceedance probability for the cross-sectional minimum of the two stock index return series; i.e., it is always true that

$$P\{\tilde{X}_1 > s, \tilde{X}_2 > s\} = P\{Z_{\min} > s\} \quad (7)$$

with  $Z_{\min} = \min(\tilde{X}_1, \tilde{X}_2)$ . The marginal tail probability at the right-hand side can now be easily calculated by making an additional assumption on the univariate tail behavior of the auxiliary variable  $Z_{\min}$ . Ledford and Tawn (1996) argue that the bivariate dependence structure is also regularly varying under fairly general conditions, just like the marginal distributions of  $X_1$  and  $X_2$ . This implies that the marginal exceedance probability (7) is of the Pareto type or

$$P\{Z_{\min} > s\} \approx l(s)s^{-\alpha}, \alpha \geq 1 \quad (8)$$

with  $s$  large ( $p$  small) and  $l(s)$  slowly varying. The tail index  $\alpha$  not only signals the tail thickness of the auxiliary variable  $Z_{\min}$  but also reflects the dependence of the original return pair  $(X_1, X_2)$  in the tail area  $[Q_1, \infty) \times [Q_2, \infty)$ . The smaller the value of  $\alpha$ , the higher the probability mass in the tail of  $Z_{\min}$  and thus also the higher the value of the joint probability in (1). This is why the inverse parameter  $\eta = 1/\alpha$  is often dubbed the tail dependence coefficient. We can now distinguish two cases in which the  $\tilde{X}_i (i = 1, 2)$  are either asymptotically dependent or independent. In the former case,  $\alpha = 1$  and

$$\lim_{s \rightarrow \infty} P\{\tilde{X}_1 > s | \tilde{X}_2 > s\} > 0$$

Stated otherwise, the conditional tail probability defined on the pair of random variables  $(X_1, X_2)$  does not vanish in the bivariate tail. Examples of asymptotically dependent random variables include the multivariate Student- $t$  distribution and the multivariate logistic distribution (see, for example, Longin and Solnik, 2001; Poon *et al.*, 2004). For asymptotic independence of the random variables ( $\alpha > 1$ ), we have that

$$\lim_{s \rightarrow \infty} P\{\tilde{X}_1 > s | \tilde{X}_2 > s\} = 0$$

Distributions that exhibit this tail behavior include the bivariate standard normal distribution or the bivariate Morgenstern distribution. For the bivariate normal with nonzero correlation coefficient  $\rho$ , the auxiliary variable's tail descent in (8) will be governed by  $\alpha = 2/(1 + \rho)$ , whereas the bivariate Morgenstern corresponds to  $\alpha = 2$ . Note that we only reach  $\alpha = 2$  for the bivariate standard normal when  $\rho = 0$ . In general, whenever the  $\tilde{X}_i (i = 1, 2)$  are fully independent,  $\alpha = 2$  and  $P\{Z_{\min} > s\} = p^2$ . But the reverse is not true; i.e., there are joint distributions with nonzero pairwise correlation but with asymptotically independent tails. The above-mentioned Morgenstern model provides an example.

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with  $R_{X_{ij}}$  the rank of element  $X_{ij}$  in the return vector  $X_i$ . Note that the ranks are divided by  $n + 1$  instead of  $n$  to prevent division by zero in equation (6).



Steps (6), (7) and (8) show that the estimation of joint probabilities like (7) can be mapped back to a univariate estimation problem. Univariate excess probabilities can be estimated by using the inverse of the previously defined quantile estimator from de Haan *et al.* (1994):

$$\hat{p}_s = \frac{m}{n} (Z_{n-m,n})^\alpha s^{-\alpha} \quad (9)$$

where the ‘tail cut-off point’  $Z_{n-m,n}$  is the  $(n - m)$ th ascending order statistic of the auxiliary variable  $Z_{\min}$ . Just as with the marginal tails, we will estimate the auxiliary variable’s tail index by means of the Hill statistic in (4).

An estimator of the co-exceedance probability  $\tau_\beta$  in (1) now easily follows by combining (9) and (4):

$$\begin{aligned} \hat{\tau}_\beta &= \frac{\hat{p}_s}{p} \\ &= \frac{m}{n} (Z_{n-m,n})^{\hat{\alpha}} s^{1-\hat{\alpha}} \end{aligned} \quad (10)$$

for large but finite  $s = 1/p$ . When the return pair exhibits asymptotic independence ( $\alpha > 1$ ), the co-exceedance probability decreases in  $s$  and eventually reaches zero if  $s \rightarrow \infty$ . On the other hand, asymptotic dependence ( $\alpha = 1$ ) implies that the probability  $\hat{\tau}_\beta$  is always bounded away from zero. However, we will not focus on the asymptotic dependence vs. independence debate and leave the tail dependence coefficient unrestricted. Moreover, Poon *et al.* (2004) already noticed that imposing asymptotic dependence if the returns are asymptotically independent might lead to severe overestimation of co-exceedance probabilities.

In the empirical application co-exceedance probabilities will be calculated either to assess the vulnerability of sectors to aggregate shocks or to measure contagion effects between sectors. However, the above estimation framework could also be used as a technique for integrated risk management. Suppose  $X_1$  and  $X_2$  stand for two open risky positions on a company’s balance sheet. The management can specify a critical loss level  $L > 0$ , which stands for the maximum aggregate loss that is allowed without running into financial distress. However, when setting maximum allowable investments ( $I_1, I_2$ ) (or trading limits) on  $(X_1, X_2)$ , one has to take into account that these risks might be dependent, even in the tails. In order to see how the co-exceedance probability for  $(X_1, X_2)$  might be useful in setting trading limits, notice that it equals the probability that the aggregate loss will be higher than  $L$ , given a large loss in one of the positions. This directly follows from the following chain of equalities:

$$\begin{aligned} \tau_\beta &\equiv P\{X_1 > Q_1(p) | X_2 > Q_2(p)\} \\ &= P\{I_1 X_1 > I_1 Q_1(p) | I_2 X_2 > I_2 Q_2(p)\} \\ &= P\{I_1 X_1 + I_2 X_2 > I_1 Q_1(p) + I_2 Q_2(p) | I_2 X_2 > I_2 Q_2(p)\} \end{aligned}$$

Positive monotonic transforms of the marginals leave the co-exceedance probability invariant, which justifies the first equality. The second equality follows from the fact that  $I_2 X_2 > I_2 Q_2(p)$  always holds because it is the conditioning event. Thus, we can add the right-hand side inequality to the left-hand side inequality without altering  $\tau_\beta$ .

If the management wants to use the co-exceedance probability  $\tau_\beta$  in order to set trading limits, it should first agree on an acceptable value of  $\tau_\beta$ . The value of the corresponding marginal significance level  $\hat{p} = 1/\hat{s}$  now directly follows by solving (10) for  $s$ . Once we know the marginal significance level  $\hat{p}$ , univariate quantiles estimates  $\hat{Q}_1(\hat{p})$  and  $\hat{Q}_2(\hat{p})$  are obtained using (3). The trading limits  $I_1$  and  $I_2$  can now be chosen such that  $I_1\hat{Q}_1 + I_2\hat{Q}_2 = L$ . Clearly an infinite number of trading limits are allowed that all render the maximum aggregate loss  $L$ .

### 2.3. Hypothesis Testing

Equality tests for estimates of the tail index  $\alpha$ , the tail quantile  $q$  or the tail dependence parameter  $\eta$  will be based on the following statistics:

$$T_\alpha = \frac{\hat{\alpha}_1 - \hat{\alpha}_2}{\text{s.e.}[\alpha_1 - \alpha_2]} \text{ or } T_\eta = \frac{\hat{\eta}_1 - \hat{\eta}_2}{\text{s.e.}[\hat{\eta}_1 - \hat{\eta}_2]}, \hat{\eta} = 1/\hat{\alpha} \quad (11)$$

and

$$T_q = \frac{\hat{q}_1(p) - \hat{q}_2(p)}{\text{s.e.}[\hat{q}_1(p) - \hat{q}_2(p)]} \quad (12)$$

with s.e. [-] denoting the standard deviation of the estimation difference. The above equality tests will be used to test for tail asymmetry (i.e., comparing lower and upper tails of the same stock index) as well as structural change with 9/11 as candidate-breakpoint. As the daily return frequency would not provide us with a number of post-9/11 extreme returns that is sufficient for a reliable application of EVT estimation and testing procedures we decided to work with half-hour returns.

The limiting distribution of (11) and (12) directly follows from the limiting behavior of  $\hat{\alpha}$  and  $\hat{q}$ . For  $m/n \rightarrow 0$  as  $m, n \rightarrow \infty$ , it has been shown that the tail index statistic  $\sqrt{m}(\hat{\alpha} - \alpha)$  and tail quantile statistic  $\sqrt{m}/\ln(m/pn)[\ln \hat{q}(p)/q(p)]$  are asymptotically normal (see Haeusler and Teugels, 1985; de Haan *et al.*, 1994). However, high-frequency equity returns typically exhibit strong nonlinear temporal dependencies (e.g., volatility clusters or GARCH effects), whereas the cited papers only established asymptotic normality under the i.i.d. assumption. More recently, however, asymptotic normality of estimators (4) and (3) has also been established in the presence of nonlinear dependencies. Asymptotic normality still holds but for higher asymptotic variances than in the i.i.d. case (see, for example, Hsing, 1991; Resnick and Stărică, 1998; Quintos *et al.*, 2001; Drees, 2002). One can safely assume that the above test statistics come sufficiently close to normality for the relatively large empirical sample sizes employed in the paper.<sup>12</sup> Because closed-form expressions for the asymptotic standard deviations in the denominators of test statistics (11)–(12) do not exist under general nonlinear time dependence, we applied a block bootstrap procedure to estimate these standard deviations. The bootstrap is performed for 1000 replications and a block length of 50.<sup>13</sup>

<sup>12</sup> We investigated the speed of convergence toward normality of both test statistics. We therefore employed the same data-generating processes as in the estimation risk study of the quantile estimator. Size distortions were found to be small for i.i.d. draws. Deviating from the i.i.d. assumption (serial correlation, stochastic volatility) only creates size distortions for persistent GARCH processes. However, upon applying these alternative rejection regions to the testing values in the empirical application a large number of testing outcomes would remain statistically significant. Details of the simulations are available upon request.

<sup>13</sup> In order to obtain an educated guess for the optimal block length, we first simulated the variance of the Hill statistic for persistent GARCH (1, 1) processes and compared this variance with the theoretical i.i.d. value  $\alpha^2$ . The variance for

The outcomes of structural change tests for  $\hat{\alpha}$  and  $\hat{q}_p$  also bear consequences for conditional tail modeling. We earlier noticed that the unconditional distribution of a GARCH (1, 1) process with conditionally normal errors can be shown to exhibit a heavy tail (see Mikosch and Starica, 2000). The latter authors derive a closed-form relation between the tail index and the parameters of the conditional variance equation. This one-to-one relation implies that structural change in the GARCH parameters should correspond to shifts in the tail index or vice versa. Moreover, it is fairly reasonable to assume that the parameters of the conditional and unconditional distribution are also related for more complex stochastic volatility dynamics, i.e., even if we do not know their closed-form relation explicitly. Because of the relationship between the parameters of the conditional and unconditional distribution, our unconditional estimation and testing approach also provides indirect evidence for time variation and asymmetries in the parameters of conditional tail models.<sup>14</sup>

One might wonder whether the quantile test (12) is not redundant because both test statistics (11)–(12) describe the same tails. However, turning back to the definition of the tail quantile in (3), it becomes obvious that tail quantile shifts or asymmetries may both be induced by shifts or asymmetries in the tail index  $\alpha$  as well as the scaling parameter  $X_{n-m}$ , whereas tail index estimators like the Hill statistic (and the resulting equality tests) are scale invariant. Thus, it might well be that tail index equality tests do not lead to rejection but that quantile equality tests do.

### 3. EMPIRICAL RESULTS

In this section we assess how frequent extreme returns in US sectoral stock indices tend to occur. In assessing these likelihoods we distinguish between extremal stock returns in isolation (conducting a purely univariate analysis) and the frequency of simultaneous sectoral stock index booms or busts (bivariate extreme value analysis). We treat rises and falls in stock market indices separately in order to identify possible asymmetries. This can be justified by the widespread use of derivatives (e.g., hedge funds with large short positions), which implies that sudden stock market rises might be as detrimental to investors' portfolios as sharp falls in the stock market. Thus, we do not only care about downside risk. Apart from conditioning on left and right tails separately, univariate tail quantiles and bivariate co-exceedance probabilities are also separately reported for pre-9/11 and post-9/11 subsamples in order to check the presence of a '9/11 effect' in the tail behavior of returns.

We start the empirical section with a short data description. Next, we investigate the univariate tail characteristics of our sectoral stock indices by reporting tail index and accompanying tail quantile estimates. Third, we report the effects of aggregate shocks on sectoral indices by means of 'tail- $\beta$ s'. We also consider co-exceedance probabilities for 'old economy' and 'new economy' stock indices. Point estimates are complemented by a number of tests on tail asymmetry and structural breaks (i.e., is there a '9/11' effect present in the tail behavior of US sectoral stock indices and are eventual asymmetry effects—if present—aggravated or diminished after 9/11?).

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dependent data is found to be approximately twice as high. Comparable variance estimates can be obtained using a block bootstrap with block lengths of 50. Further details of the Monte Carlo simulation are available upon request.

<sup>14</sup> One could also try to perform direct tests for structural change and asymmetry on the parameters governing the conditional distribution. However, the high-frequency character of our data complicates the matter because one should also take into account intraday volatility seasonalities (see, for example, Andersen and Bollerslev, 1997; Bollerslev *et al.*, 2000).

### 3.1. Data Description

We collected half-hourly stock price data for 19 US sectoral stock market price indices. Returns were calculated as log price differences. Overnight and weekend returns were linearly rescaled to the half-hour time horizon.<sup>15</sup> The sectoral stock indices will be listed using the following abbreviations: Dow Industrials (IND), Dow Transport (TRAN), Dow Utilities (UTIL), Nasdaq Computers (PC), Nasdaq Biotechnology (BIO), Nasdaq Insurance (INSUR), Nasdaq Telecom (TEL), Nasdaq Banking (BANK), Nasdaq Finance (FIN), Nasdaq Other Finance (OFIN), internet (INTER), oil (OIL), Pharmaceuticals (PHARMA), Airlines (AIR), NYSE Composite (NYCOMP), S&P Smallcap 600 (SCAP), S&P Midcap 400 (MCAP), S&P/BARRA growth (GROWTH) and S&P/BARRA value (VALUE) indices. All series start on 18 February 1999 and end on 15 April 2004, rendering 16,761 return observations per index. High-frequency data were obtained via the download program Qcharts from lycos.com. Qcharts provides an interface to Wall Street and the Chicago Board of Exchange (CBOE) and offers both online price information and historical time series data.

We calculated pre-9/11 and post-9/11 descriptive return statistics such as mean, standard deviation, skewness and kurtosis. Average returns are basically zero, as one would expect on such high data frequencies. Not surprisingly, new technology stocks (biotech, Internet, telecom, computers) exhibit the highest standard deviations. Contrary to what one would expect, however, the standard deviations only rise after 9/11 in a minority of cases. While there are little signs of skewness in the pre-9/11 sample, the skewness parameter declines and becomes negative for a majority of indices in the post-9/11 sample. Finally, the high kurtosis signals that all series are highly leptokurtic. Excess kurtosis increased after 9/11 for a majority of the indices.

For the purpose of the present paper, we are particularly interested in extreme negative and positive returns. Table I reports the two most extreme returns in the upper and lower empirical tails for our sample of 19 intraday US stock index returns. We further distinguish between pre-9/11 extremes and post-9/11 extremes. Corresponding calendar dates are reported in parentheses below each return.

The table enables one to compare the magnitude and timing of extremes across sectors, time and lower/upper tails. We observe quite a lot of cross-sectoral heterogeneity in the tail extremes irrespective of the time period considered. Return tails seem to be wider in the pre-9/11 period for technology stocks (computers, telecom, Internet, biotechnology) while extreme losses or gains for the other indices are of comparable magnitude (except post-9/11 airline and transportation index returns). Notice also that the midcap vs. smallcap extremes and growth vs. value extremal returns barely differ. This suggests that growth, value and size effects are relatively absent in periods of market turbulence. Somewhat surprisingly, post-9/11 historical lows are most often dominated by the pre-9/11 historical minima (except transport, airlines, oil and utilities). Also surprising is that historical extreme losses do not seem to exceed extreme gains in either of the two subsamples.<sup>16</sup>

Apart from providing a lot of preliminary univariate information, the calendar dates in the table also offer some first evidence of clustering in extreme sectoral stock market returns. Part of the

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<sup>15</sup> We did not have half-hourly data of 3-month T-bills in order to calculate co-exceedance probabilities for excess returns. However, tail- $\beta$ s hardly change when conditioned on excess returns instead of ordinary returns; see Hartmann *et al.* (2005) for an example with daily data.

<sup>16</sup> This seems to contradict earlier studies of the empirical distribution function of stock returns that do find a dominance of left tail extremes over right tail extremes (see, for example, Jansen and de Vries, 1991; Hartmann *et al.*, 2004). The difference in results with our paper is probably due to the high-frequency character of our data and the much shorter sample period in our study.

Table I. US historical extreme returns

Indices	<9/11				>9/11			
	$X_{1,n}$ (Date)	$X_{2,n}$ (Date)	$X_{n-1,n}$ (Date)	$X_{n,n}$ (Date)	$X_{1,n}$ (Date)	$X_{2,n}$ (Date)	$X_{n-1,n}$ (Date)	$X_{n,n}$ (Date)
IND	-3.08 (4/4/0)	-2.02 (3/7/0)	1.95 (4/18/1)	3.44 (1/3/1)	-1.96 (7/22/2)	-1.55 (7/19/2)	2.19 (7/15/2)	2.27 (7/22/2)
TRAN	-1.64 (12/14/0)	-1.45 (6/2/99)	1.77 (1/3/1)	1.84 (3/23/00)	-10.97 (9/17/1)	-2.07 (10/29/2)	2.03 (9/21/1)	2.16 (10/3/0)
UTIL	-3.26 (4/6/1)	-3.02 (1/4/1)	2.04 (4/4/0)	2.13 (4/6/1)	-3.63 (10/8/2)	-2.89 (7/15/2)	3.80 (10/8/2)	4.26 (10/10/2)
PC	-4.13 (4/4/0)	-3.92 (4/4/0)	4.90 (4/4/0)	8.23 (1/3/1)	-2.73 (10/17/1)	-2.47 (7/25/2)	3.27 (10/3/1)	3.33 (9/19/1)
BIO	-7.20 (3/14/0)	-4.52 (3/14/0)	4.51 (4/5/0)	7.95 (1/3/1)	-3.19 (6/6/3)	-2.68 (7/2/2)	3.29 (5/23/2)	3.56 (6/14/2)
INSUR	-4.35 (3/16/99)	-2.29 (4/14/99)	2.75 (3/16/99)	2.92 (4/12/99)	-1.19 (7/22/2)	-1.13 (9/3/2)	2.04 (7/25/2)	2.31 (9/17/1)
TEL	-4.81 (4/4/0)	-4.12 (4/4/0)	5.44 (4/4/0)	6.43 (1/3/1)	-2.80 (7/25/2)	-2.56 (6/27/2)	3.00 (7/15/2)	3.14 (10/10/2)
BANK	-1.49 (4/4/0)	-1.20 (1/3/0)	1.70 (3/21/0)	2.28 (1/3/1)	-1.11 (7/23/2)	-0.97 (9/21/1)	1.54 (7/24/2)	1.71 (7/25/2)
FIN	-2.18 (4/4/0)	-1.50 (4/4/0)	1.84 (3/21/0)	2.72 (1/3/1)	-1.31 (7/23/2)	-1.03 (9/21/1)	1.76 (10/10/2)	2.04 (7/25/2)
OFIN	-6.20 (4/15/99)	-4.92 (4/4/0)	3.98 (4/4/0)	4.49 (4/15/99)	-1.48 (1/21/3)	-1.47 (8/13/2)	1.68 (10/3/1)	2.29 (9/21/1)
INTER	-8.95 (6/19/0)	-5.37 (4/4/0)	6.14 (4/4/0)	10.39 (1/3/1)	-3.15 (7/25/2)	-3.14 (10/17/1)	3.51 (10/3/1)	3.75 (9/19/1)
PHARMA	-2.47 (1/3/1)	-2.21 (8/9/0)	1.57 (1/7/0)	2.16 (8/23/99)	-1.99 (5/19/3)	-1.79 (7/22/2)	2.15 (8/6/2)	2.43 (7/15/2)
AIR	-2.22 (1/9/1)	-2.00 (12/14/0)	2.39 (3/23/0)	2.86 (9/9/99)	-20.21 (9/17/1)	-12.18 (9/17/1)	3.59 (10/22/2)	3.75 (11/6/2)
OIL	-1.63 (1/3/1)	-1.57 (3/8/0)	1.72 (3/2/0)	1.75 (2/10/0)	-2.20 (7/22/2)	-1.84 (10/24/2)	1.66 (8/9/2)	1.92 (8/6/2)
SCAP	-1.77 (4/4/0)	-1.76 (4/4/0)	2.09 (4/17/0)	2.11 (4/4/0)	-1.20 (6/27/2)	-1.16 (11/26/2)	1.77 (1/27/3)	1.82 (9/19/1)
MCAP	-2.32 (4/4/0)	-2.04 (4/4/0)	2.25 (4/4/0)	2.87 (1/3/1)	-1.45 (7/22/2)	-1.20 (6/27/2)	1.63 (9/19/1)	1.80 (10/10/2)
GROWTH	-2.91 (4/4/0)	-2.55 (4/4/0)	2.93 (4/4/0)	3.78 (1/3/1)	-1.89 (7/22/2)	-1.54 (7/19/2)	2.05 (7/15/2)	2.06 (10/10/2)
VALUE	-2.16 (4/4/0)	-1.56 (1/11/1)	1.81 (10/28/99)	3.25 (1/3/1)	-1.86 (7/23/2)	-1.53 (7/22/2)	2.32 (7/25/2)	2.33 (10/10/2)
NYCOMP	-2.42 (4/4/0)	-1.50 (4/4/0)	1.53 (4/4/0)	3.09 (3/1/1)	-1.48 (7/22/2)	-1.45 (7/23/2)	2.00 (7/25/2)	2.05 (7/15/2)

Note:  $X_{1,n}$  and  $X_{2,n}$  are the two smallest intradaily (half-hour) historical returns, whereas  $X_{n-1,n}$  and  $X_{n,n}$  stand for the two highest observed returns. Dates in parentheses are denoted XX/YY/ZZ, where XX = month, YY = day and ZZ = year. Data are from 18 February 1999 to 15 April 2004.

negative extremes in spring and summer of 2000 can potentially be explained by the burst of the technology bubble. A number of extremes can also be linked to monetary policy announcements. For example, the positive extremal returns in early January 2001 were probably due to a decrease in the federal funds rate and discount rate around that time.<sup>17</sup> Also, notice that the historical rises

<sup>17</sup> On 3–4 January 2001 the Federal Open Market Committee decided to lower its target for the federal funds rate by 50 basis points. In line with this decision, the Board of Governors approved a 25-basis-point decrease in the discount rate.

during mid-October 2002 overlap with discussions in the American Congress on the precise content of the Terrorism Risk Insurance Act (TRIA).<sup>18</sup> Finally, a lot of historical extremes cluster together in short intervals, which indicates that half-hourly returns exhibit strong temporal dependencies, even in the extremes. We now turn to a more rigorous investigation of extreme return occurrence around 9/11.

### 3.2. Univariate Results

In Figure 1 we report tail index estimates (top panel graph) and accompanying tail quantiles (bottom panel graph) for our sample of US stock indices. Estimates are separately reported for left and right tails.

We further distinguish between pre-9/11 and post-9/11 subsample estimates. The Hill statistics cum quantile estimates are conditioned on optimal nuisance parameters  $m$  determined with the Beirlant *et al.* (1999) algorithm.<sup>19</sup> In line with previous studies, the tail index is found to be relatively stable across sectors, time periods and across upper and lower tails. It fluctuates around 3, reaches a minimal value of 2.24 for the left tail (post-9/11) of the utility index whereas a maximum value of 5.87 is reached by the lower tail (post-9/11) of the midcap index. This cross-sectional homogeneity in tail index estimates already suggests that the tail index alone cannot be a good measure of sectoral tail risk. The estimates further illustrate the non-normality of sectoral stock index returns and the non-boundedness of higher moments. We find that right tail indices are often smaller than left tail indices for both the pre-9/11 and post-9/11 period. This suggests that there is more upward potential than downside risk. Even more surprising is the observation that both the upper and lower tail index seems to increase in a large number of cases in the aftermath of 9/11.

The economic issue of interest, however, is to use the tail index estimates in order to assess the ‘downside risk’ or ‘upward potential’ for the sectoral indices considered by means of left and right tail quantile estimates. Risk managers might be interested in assessing the likelihood of occurrence of large-scale losses or gains in order to calculate capital requirements or trading limits for risky open positions (see, for example, Danielsson and de Vries, 1997). The graphs in the lower panel of Figure 1 report estimated quantiles using (3) for all the considered indices.<sup>20</sup> We experiment with values for the common significance level  $p$  equal to 0.02% because the implied boom or bust levels tend to be close to the endpoint of the historical sample boundaries described in Table I. Thus, there cannot be much doubt that the price increases or falls corresponding to this marginal significance level constitute intradaily stress situations for US sectoral investors and portfolio managers. Turning to the economic interpretation of a tail quantile, notice that the inverse of a quantile’s significance level  $p$  is the expected waiting time or time span for an extreme event of the estimated quantile magnitudes to happen. For example, the (pre-9/11) 1.79% half-hourly

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<sup>18</sup> A large part of intraday extremes and their co-occurrence remains unexplained. In a previous study, Fair (2002) also experienced difficulty in linking high-frequency S&P 500 return extremes to news events such as monetary policy announcements.

<sup>19</sup> We restricted the optimal Beirlant *et al.* values to be at least equal to 1% in the univariate case. The constraint is only binding in a limited number of cases.

<sup>20</sup> Due to the strong nonlinear temporal dependencies in the return data on the intradaily frequency, scaling laws for scaling up the VaR levels from an intradaily to a daily or weekly time horizon are problematic to implement. We therefore leave the quantile estimates’ time horizon equal to the data frequency.

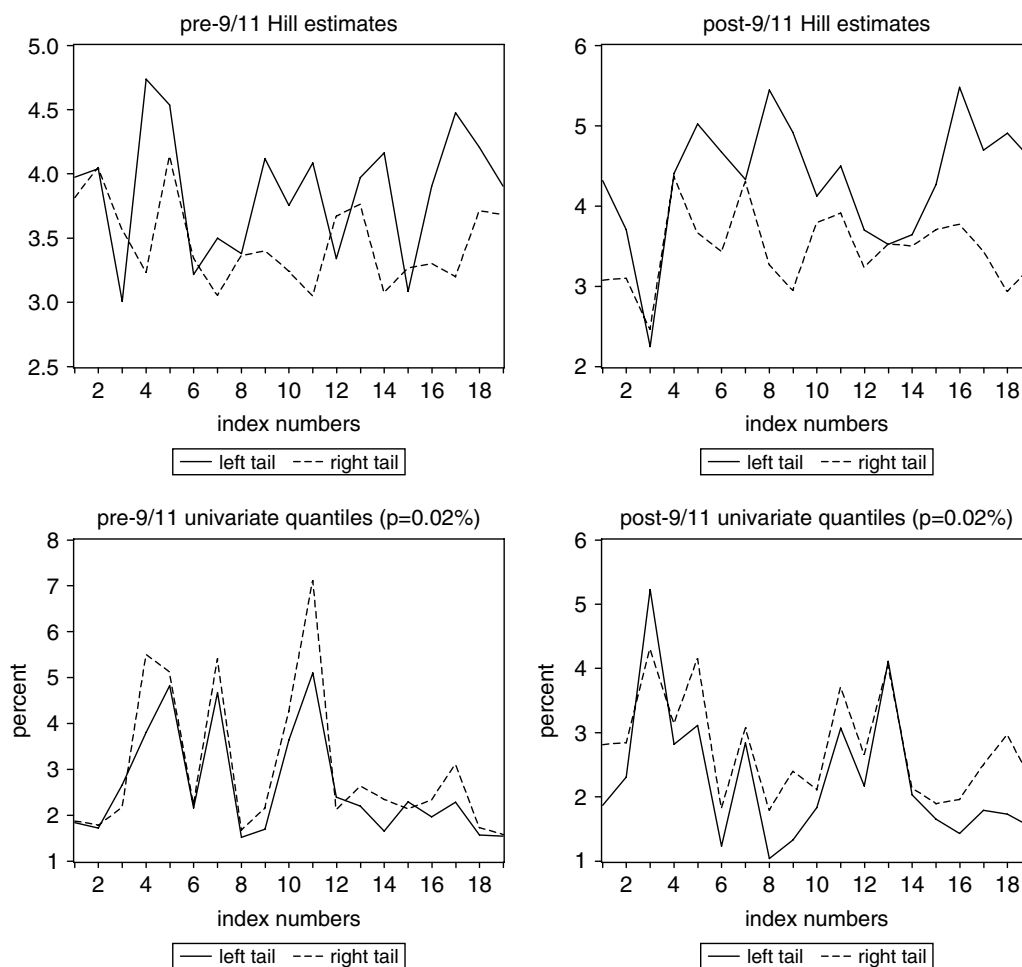


Figure 1. Tail index and quantile estimates for sectoral indices. *Note:* The numbers on the horizontal axis correspond to the following indices (abbreviated): IND(1), TRAN(2), UTIL(3), PC(4), BIO(5), INSUR(6), TEL(7), BANK(8), FIN(9), OFIN(10), INT(11), PHARMA(12), AIR(13), OIL(14), SCAP(15), MCAP(16), GROWTH(17), VALUE(18), NYCOMP(19)

slump of the Dow Jones Industrial (left tail quantile that corresponds with  $p = 0.02\%$ ) is expected to happen roughly once every one and a half years.<sup>21</sup>

Upon comparing the quantile magnitudes across sectors, time periods and upper/lower tails, the estimates exhibit more heterogeneity than the tail indices on which the quantile estimates are conditioned. This larger dispersion in quantile estimates can be explained by cross-sectoral differences in the scaling variable  $X_{n-m,n}$ . The quantile magnitudes are found to be highest for 'new technology' indices compared to more traditional 'old technology' indices, which confirms

<sup>21</sup> The inverse of the significance level amounts in this case to  $1/0.0002 = 5000$  trading half-hours. With 13 trading half-hours in a trading day and 260 trading days in a year, a year consists of 3380 trading half-hours. Thus, the expected waiting time for the half-hour slump in DJIA amounts to  $5000/3380 \approx 1.48$  years.

the high standard deviations and historical extremes (Table I) for these sectors. Moreover, and in line with the tail index results, we find that post-9/11 quantile estimates only exceed their pre-9/11 counterparts in a minority of cases. Also, left tail quantiles (reflecting downside risk) only dominate right tail quantiles (reflecting ‘upward potential’) in a minority of cases.

We assessed the statistical significance of these corroborated asymmetries and/or time variation in Hill statistics and corresponding quantile estimates by means of test statistics (11) and (12). Testing results are reported in Table II. The table’s left and right panels report values of the null hypothesis of tail index constancy  $T_\alpha$  and tail quantile constancy  $T_q$ , respectively. We further distinguish between asymmetry tests and structural change tests. The structural change tests on  $\alpha$  and  $q$  assess the statistical significance of the differences  $\hat{\alpha}(< 9/11) - \hat{\alpha}(> 9/11)$  and  $\hat{q}(< 9/11) - \hat{q}(> 9/11)$ , respectively, and this for the left and right tails separately. Increases in left and right tail risk over time correspond to significantly positive values of (11) and significantly negative values of (12). The asymmetry tests, on the other hand, reflect whether the differences  $\hat{\alpha}(left) - \hat{\alpha}(right)$  and  $\hat{q}(left) - \hat{q}(right)$  are statistically significant and this for both the pre-9/11 and post-9/11 subsamples separately.

Turning to the testing results in Table II, structural change and asymmetry in tail indices and tail quantiles are clearly non-negligible (although tail asymmetries seem to occur less often). Moreover, it is striking to see that structural change most often corresponds to rising tail indices (falling tail quantiles) for both upper and lower tails, whereas one would have expected the reverse to happen

Table II. Structural change/asymmetry tests for tail indices and univariate quantiles

Indices	$T_\alpha[H_0: \alpha_1 = \alpha_2]$				$T_q[H_0: q_1(p) = q_2(p)]$			
	Structural change		Asymmetry		Structural change		Asymmetry	
	$l_1 = l_2$	$r_1 = r_2$	$l_1 = r_1$	$l_2 = r_2$	$l_1 = l_2$	$r_1 = r_2$	$l_1 = r_1$	$l_2 = r_2$
IND	-0.599	2.010**	0.417	2.719***	-0.102	-2.338***	-0.155	-3.193***
TRAN	0.813	2.282**	-0.019	1.708*	-2.148**	-3.120***	-0.224	-1.525
UTIL	2.461***	2.199**	-1.347	-1.337	-2.453***	-2.681***	1.576	1.459
PC	0.525	-2.284**	2.811***	0.086	2.211**	3.601***	-3.094***	-0.975
BIO	-0.656	0.875	0.598	2.305***	3.271***	1.417	-0.427	-2.356***
INSUR	-2.364**	-0.168	-0.342	1.742*	2.998***	1.068	-0.218	-2.104**
TEL	-2.183**	-2.767***	1.871*	0.056	3.924***	3.509***	-1.229	-0.646
BANK	-3.209***	0.217	0.048	3.193***	2.889***	-0.311	-0.587	-3.349***
FIN	-1.394	1.189	1.569	3.852***	1.947*	-0.525	-1.675	-3.841***
OFIN	-0.694	-1.141	1.060	0.754	4.087***	3.879***	-1.143	-1.159
INTER	-0.724	-2.188**	2.442	1.306***	3.426***	3.968***	-2.828***	-1.631
PHARMA	-0.850	1.284	-1.107	1.020	0.594	-1.523	1.004	-1.243
AIR	1.136	0.543	0.454	-0.010	-5.205	-2.849	-1.245	0.104
OIL	0.944	-1.289	2.918***	0.332	-1.117	0.634	-2.660***	-0.339
SCAP	-3.131***	-0.987	-0.618	1.270	2.108**	0.619	0.564	-1.031
MCAP	-2.443***	-0.988	1.573	2.705	2.053**	0.981	-1.341	-2.260**
GROWTH	-0.321	-0.736	2.634***	2.280**	1.657	1.359	-2.736***	-2.449***
VALUE	-0.900	1.552	0.930	3.336***	-0.564	-2.836***	-0.683	-4.448***
NYCOMP	-1.170	0.867	0.489	2.731***	0.039	-1.848*	-0.172	-2.943***

Note: The tests for tail index and tail quantile equality are defined in equations (11) and (12), respectively. Structural change is separately tested for the left ( $l_1 = l_2$ ) and right ( $r_1 = r_2$ ) tail, whereas left tail–right tail asymmetry is tested for the pre-9/11 ( $l_1 = r_1$ ) and post-9/11 ( $l_2 = r_2$ ) subsample. The equal quantiles test is conditioned upon  $p = 0.02\%$ . The test is asymptotically normal in large samples and two-sided rejections at the 10%, 5% and 2% significance level are denoted by \*, \*\* and \*\*\*, respectively.



as a consequence of 9/11 (more downside risk). Also, the statistically significant tail asymmetries correspond to upper tails that dominate lower tails; this seems to contradict some earlier work in EVT.<sup>22</sup>

### 3.3. Bivariate Results

In this section we present and interpret the estimation results of our co-exceedance probability defined in (1). We will use it both as a measure of extreme systematic risk or ‘tail- $\beta$ ’ as well as a bilateral linkage measure for pairs of sectoral portfolios. In the latter case the conditioning portfolio in equation (2.1) is one of the sectoral portfolios, whereas tail- $\beta$ s reflect the sensitivity of sectoral stock indices to ‘aggregate’ shocks as captured by extreme fluctuations in macro factors. As macro shock transmitters, we decided to select extreme movements in a market risk factor (NYSE Composite), as in traditional asset pricing theory, and an oil index.<sup>23</sup> As concerns direct sectoral bilateral linkages we limited ourselves to investigating the extreme linkages within and between the old and new technology stock indices.

Estimation results on tail dependence parameters and accompanying co-exceedance probabilities are summarized in Figures 2 and 3 (tail- $\beta$ s) and Figure 4 (bilateral linkage results), respectively.<sup>24</sup> The tail- $\beta$ s are conditioned on the NYSE Composite (Figure 2) and the oil index (Figure 3). All three figures consist of an upper panel of two graphs with estimates of the tail dependence parameter  $\eta$  and a lower panel of two graphs with co-exceedance probabilities. The tail dependence coefficient  $\eta$  is calculated by means of the Hill statistic as defined in equation (4), whereas the conditional probability estimates  $\hat{\beta}_\tau$  correspond to (10). The graphs distinguish between pre-9/11 and post-9/11 subsamples and lower (3rd data quadrant) and upper (1st data quadrant) bivariate tails. All co-exceedance probabilities are evaluated for a marginal significance level  $p = 0.02\%$ . The value of  $p$  determines how deep we go into the bivariate tail (lower values of  $p$  imply higher values of the marginal quantiles  $Q_1(p)$  and  $Q_2(p)$  in equation (10) and thus more ‘extreme’ co-exceedance probabilities). Figure 4 reports co-exceedance probabilities for pairs of old economy indices (1st segment), new economy indices (3rd segment) and mixed pairs of old/new economy indices (middle segment).

Because the bivariate estimation problem can be reduced to a univariate estimation of the tail of an auxiliary variable  $Z_{\min}$ , we again use the Beirlant *et al.* (1999) algorithm for selecting the nuisance parameter  $m$  in equations (4) and (10). The bivariate threshold values are found to be much higher than their counterparts for the tails of the marginal distributions. This reflects that the univariate tails of the raw returns are thinner than the auxiliary variable’s tail  $Z_{\min}$ ; i.e., a heavier tail implies that more extremes can be used in estimation.

Tail dependence parameter estimates  $\hat{\eta}$  all lie way above 0.5 but still below 1; this corresponds to values for the tail index  $\hat{\alpha} = 1/\hat{\eta}$  between 1 and 2. Thus the tail of the auxiliary variable  $Z_{\min}$  as defined in (7) contains more probability mass than the tails of the original return series indeed (see the tail index estimates in the upper panel of Figure 1). Although we did not explicitly

<sup>22</sup> Observed asymmetry between the historical minimum and maximum returns have also been reported in de Haan *et al.* (1994), Longin and Solnik (2001), Hartmann *et al.* (2004) and Jondeau and Rockinger (2003). Notice, however, that all these studies work with daily data. In contrast to our results, these previous studies typically find that lower tails are heavier than upper tails, albeit the statistical significance is usually small.

<sup>23</sup> One might think of yet other conditioning factors. For example, tail- $\beta$ s for bank stocks conditioned on high-yield bond spreads have been considered in Hartmann *et al.* (2005) and were found to be surprisingly small.

<sup>24</sup> All point estimates from the graphs are available upon request from the authors but are omitted for space considerations.

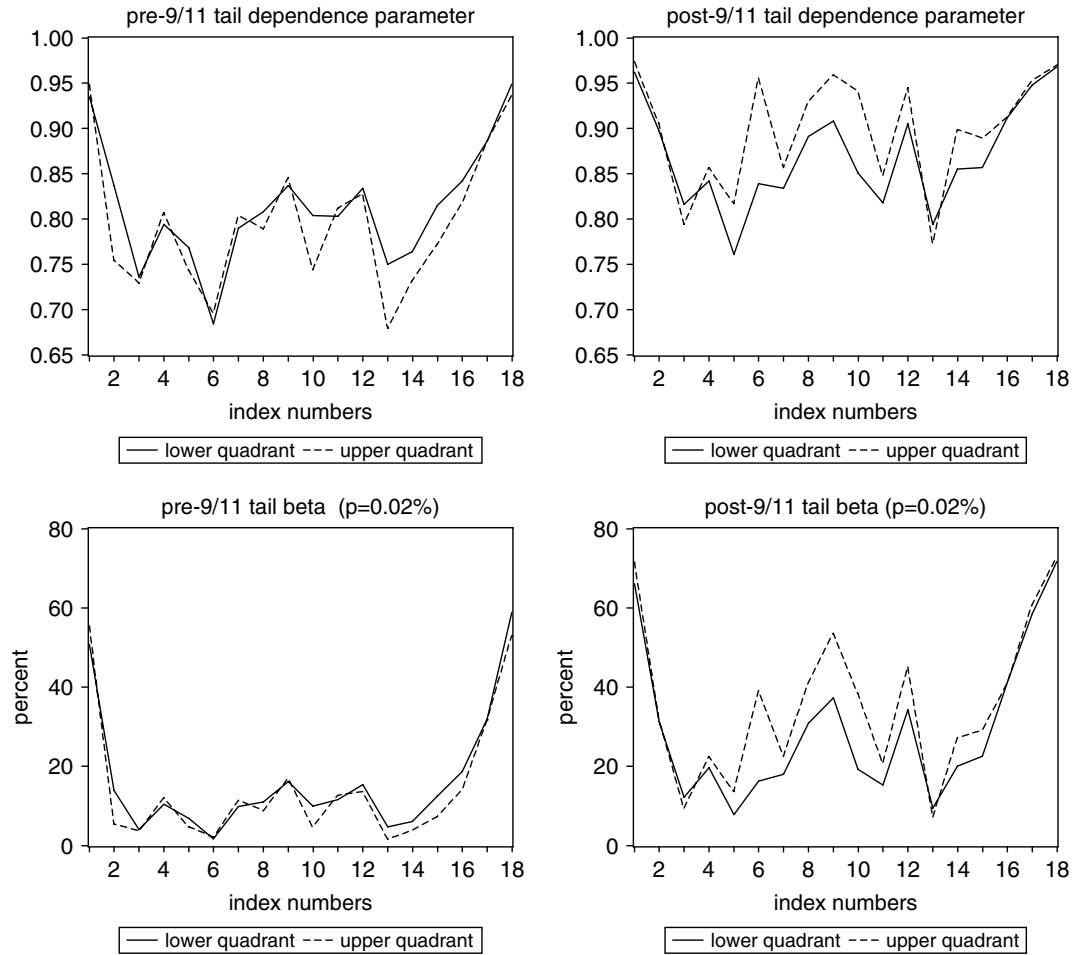


Figure 2. Tail dependence parameters and tail- $\beta$ s for sectoral indices w.r.t. NYSE Composite. *Note:* The numbers on the horizontal axis correspond to the following indices (abbreviated): IND (1), TRAN (2), UTIL (3), PC (4), BIO (5), INSUR (6), TEL (7), BANK (8), FIN (9), OFIN (10), INT (11), PHARMA (12), AIR (13), OIL (14), SCAP (15), MCAP (16), GROWTH (17), VALUE (18)

test the null hypothesis of complete independence ( $H_0: \eta = 1/2$ ), most of the tail dependence estimates exceed 0.7 which suggests the presence of positive dependence between sectoral returns and between returns and common factors like the NYSE Composite. The figures also show that higher values of  $\eta$  usually imply higher co-exceedance probabilities.

Co-exceedance probabilities have a natural economic interpretation. For example, the 10.42% tail- $\beta$  for the pre-9/11 NASDAQ Computer index (lower tail) in Figure 2 means that once a ‘large’ half-hourly downturn in the NYSE Composite strikes then this event is expected to coincide with an ‘extreme’ decline in the Computer index during 10.42% of the time, i.e., on average every  $1/0.1042 \approx 10$  half-hours. The ‘large’ downturns are the 0.02% left tail quantiles for the NYSE Composite and the Computer index in the lower panel of Figure 1 (1.54% and 3.81%, respectively).

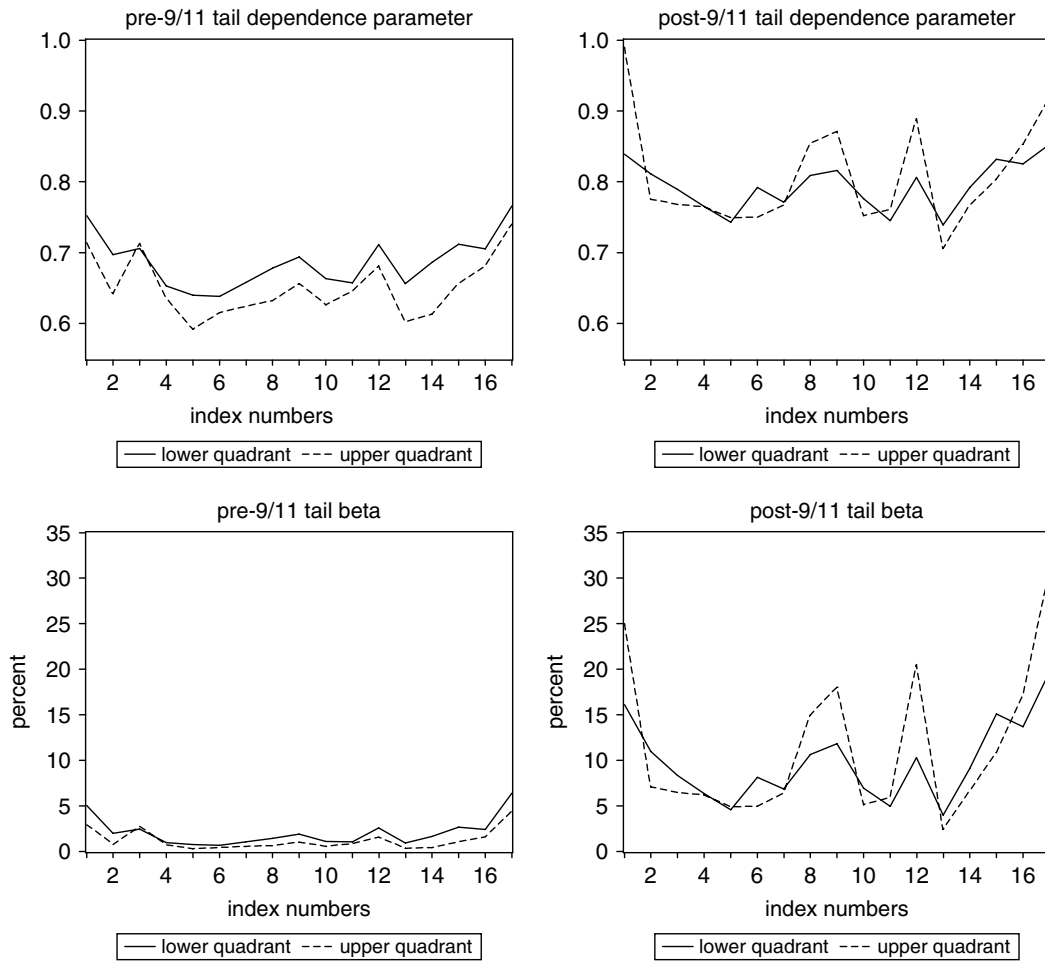


Figure 3. Tail dependence parameters and tail- $\beta$ s for sectoral indices w.r.t. oil index. *Note:* The numbers on the horizontal axis correspond to the following indices (abbreviated): IND (1), TRAN (2), UTIL (3), PC (4), BIO (5), INSUR (6), TEL (7), BANK (8), FIN (9), OFIN (10), INT (11), PHARMA (12), AIR (13), SCAP (14), MCAP (15), GROWTH (16), VALUE (17)

We found a value of 19.60% for the post-9/11 tail- $\beta$  (lower tail); this indicates that the Computer index is much more likely to co-crash with the market since 9/11.

In order to put the magnitude of the tail- $\beta$ s better into perspective, one has to compare them with the marginal probability of experiencing a crash in one sectoral index at the time. This is the marginal significance level 0.02% on which co-exceedance probabilities are conditioned. Clearly, the (conditional) probability of experiencing an extreme event in a sectoral index given there is already one in another market (the market risk factor or the oil index) is markedly higher than the likelihood of extremal events 'in isolation', i.e., without using conditioning information. This illustrates the relevance of phenomena like contagion or joint crises as a consequence of a common shock. In other words, while severe security market crises are fairly rare events if one predicts

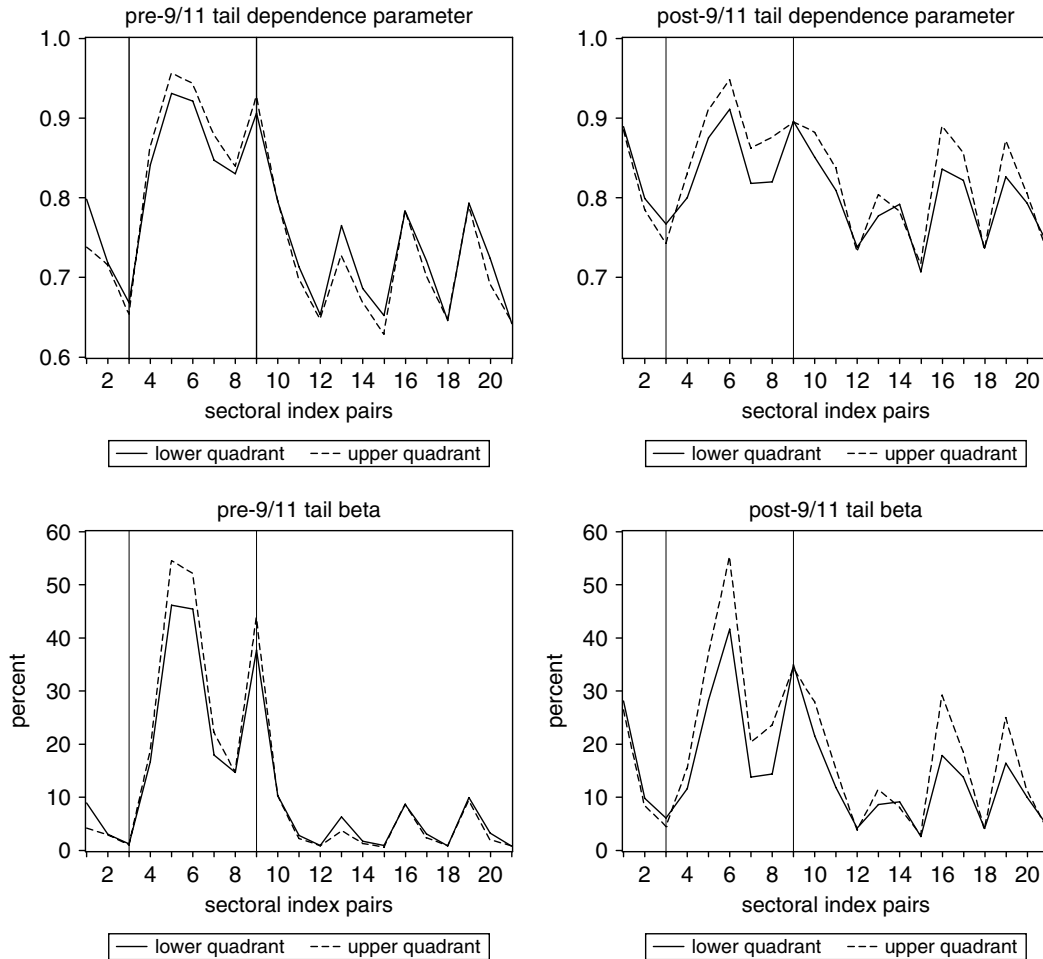


Figure 4. Tail dependence parameters and co-exceedance probabilities for sectoral indices. *Note:* The numbers on the horizontal axis correspond to the following index pairs (abbreviated): IND-TRAN (1), IND-UTIL (2), TRAN-UTIL (3), PC-BIO (4), PC-TEL (5), PC-INTER (6), BIO-TEL (7), BIO-INTER (8), TEL-INTER (9), IND-PC (10), TRAN-PC (11), UTIL-PC (12), INDBIO (13), TRAN-BIO (14), UTIL-BIO (15), IND-TEL (16), TRAN-TEL (17), UTIL-TEL (18), INDINTER (19), TRAN-INTER (20), UTIL-INTER (21). Old economy linkages (left panel), new economy linkages (middle panel) and mixed old–new economy linkages (right panel) are separated from each other by vertical lines

them without using price information from other markets, it is not that unlikely for sudden booms or busts to occur jointly once one market is hit by a sharp rise or drop. The higher values of the conditional probabilities compared to the marginal significance level  $p$  are due to the fact that sectoral stock market indices and the conditioning factors exhibit pairwise dependence, i.e.,  $\hat{\eta} > 0.5$  and as a result  $\hat{\beta}_\tau > p$ .

Upon comparing the magnitudes of the co-exceedance probabilities in the figures, the oil index tail- $\beta$ s in Figure 3 are found to be much smaller than the NYSE Composite tail- $\beta$ s in Figure 2. This is not too surprising because the oil factor is not spanning global market movements. As concerns

the magnitude of the co-exceedance probabilities for sector pairs in Figure 4, ‘new economy’ index pairs seem to be more strongly interlinked during crisis periods than either pairs of old economy indices (first segment in Figure 4) or mixed pairs of old/new economy indices (middle segment in Figure 4). Also, tail dependence parameters and co-exceedance probabilities increase after 9/11 in nearly all cases (most strongly for oil index tail- $\beta$ s). Finally, notice that the above graphs are not suggestive of strong asymmetries in the co-exceedance probabilities (compare full and dotted lines in graphs).

The tail- $\beta$  estimates for the NYSE Composite and the oil index weakly suggest that sectors are more prone to co-crashes prior to 9/11 but that co-booms become more likely afterwards. As for the intersectoral co-exceedance probabilities in Figure 4, there does not seem to be any graphical evidence at all for tail asymmetries.

It remains to be seen whether the above structural change and asymmetry corroborations survive statistical testing. We therefore implemented a pair of tests for the null hypothesis of tail dependence constancy ( $H_0: \eta_1 = \eta_2$ ) and co-exceedance probability constancy ( $H_0: \tau_{\beta_1} = \tau_{\beta_2}$ ). Tables III, IV and V report results of structural change and asymmetry tests for NYSE tail- $\beta$ s, oil index tail- $\beta$ s and sectoral co-exceedance probabilities, respectively. Notice these are the same type of tests as in Table II (univariate tail behavior). The null hypothesis of constancy either takes the form of constancy over time (absence of structural change) or constancy across the lower and upper tails (absence of asymmetry). The structural change tests for  $\eta$  and  $\tau_\beta$  assess the

Table III. Structural change/asymmetry tests for tail dependence parameters and tail betas w.r.t. NYSE Composite

Index	$T_\eta[H_0: \eta_1 = \eta_2]$				$T_q[H_0: \beta_1(p) = \beta_2(p)]$			
	Structural change		Asymmetry		Structural change		Asymmetry	
	$l_1 = l_2$	$u_1 = u_2$	$l_1 = u_1$	$l_2 = u_2$	$l_1 = l_2$	$u_1 = u_2$	$l_1 = u_1$	$l_2 = u_2$
IND	-0.599	-0.461	-0.465	-0.395	-0.643	-0.575	-0.346	-0.338
TRAN	-1.229	-3.055***	2.402**	-0.187	-1.728*	-3.299***	2.297**	-0.039
UTIL	-1.932*	-1.666*	0.340	0.999	-2.062**	-1.834*	0.305	0.956
PC	-1.190	-1.114	-0.592	-0.604	-1.339	-1.266	-0.542	-0.526
BIO	0.160	-1.790**	1.202	-1.623	-0.320	-2.013**	1.287	-1.439
INSUR	-4.502***	-3.534***	-0.616	-1.943*	-4.877***	-3.892***	-0.618	-1.746*
TEL	-1.039	-1.167	-0.556	-0.894	-1.234	-1.345	-0.481	-0.825
BANK	-2.129**	-2.616***	0.801	-1.051	-2.425***	-2.804***	0.780	-0.885
FIN	-1.760*	-1.894*	-0.336	-1.324	-2.015**	-2.130**	-0.131	-1.140
OFIN	-1.067	-2.792***	2.185**	-1.618	-1.383	-2.986***	2.112**	-1.468
INTER	-0.378	-0.846	-0.423	-1.175	-0.615	-1.055	-0.331	-1.086
PHARMA	-1.663*	-1.881*	0.250	-0.989	-1.826*	-2.057**	0.467	-0.830
AIR	-1.356	-2.984***	2.976***	0.884	-1.628	-3.232***	2.875***	0.921
OIL	-2.377***	-3.012***	1.466	-1.026	-2.575***	-3.188***	1.441	-0.760
SCAP	-1.047	-2.094**	1.884*	-0.880	-1.362	-2.321**	1.826*	-0.724
MCAP	-1.704*	-2.020**	0.979	-0.035	-1.898*	-2.216**	1.022	0.034
GROWTH	-1.420	-1.372	0.023	-0.201	-1.514	-1.465	0.029	-0.166
VALUE	-0.394	-0.696	0.459	-0.085	-0.500	-0.772	0.433	-0.067

Note: The tests for tail dependence and tail beta equality are defined in equations (11) and (12), respectively. Structural change is separately tested for the lower ( $l_1 = l_2$ ) and upper ( $u_1 = u_2$ ) bivariate tail whereas lower tail–upper tail asymmetry is tested for the pre-9/11 ( $l_1 = u_1$ ) and post-9/11 ( $l_2 = u_2$ ) subsample. The equal tail beta test is conditioned upon  $p = 0.02\%$ . The test is asymptotically normal in large samples and two-sided rejections at the 10%, 5% and 2% significance level are denoted by \*, \*\*, and \*\*\*, respectively.

Table IV. Structural change/asymmetry tests for tail dependence parameters and tail betas w.r.t. oil index portfolio

Indices	$T_\eta[H_0: \eta_1 = \eta_2]$				$T_q[H_0: \beta_1(p) = \beta_2(p)]$			
	Structural change		Asymmetry		Structural change		Asymmetry	
	$l_1 = l_2$	$u_1 = u_2$	$l_1 = u_1$	$l_2 = u_2$	$l_1 = l_2$	$u_1 = u_2$	$l_1 = u_1$	$l_2 = u_2$
IND	-2.352***	-2.995***	1.790*	-1.279	-2.534***	-3.175***	1.703*	-0.969
TRAN	-3.151***	-4.167***	2.657***	1.434	-3.468***	-4.501***	2.519***	1.352
UTIL	-1.953*	-1.614	-0.357	0.824	-2.104**	-1.817*	-0.343	0.776
PC	-3.819***	-4.267***	0.996	0.086	-4.121***	-4.673***	0.991	0.111
BIO	-3.393***	-4.959***	2.636***	-0.268	-3.676***	-5.353***	2.600***	-0.189
INSUR	-3.610***	-4.171***	1.234	1.274	-4.018***	-4.604***	1.206	1.158
TEL	-3.653***	-4.960***	1.792	0.241	-3.941***	-5.340***	1.732*	0.264
BANK	-3.924***	-3.674***	2.132**	-0.929	-4.258***	-4.014***	2.021**	-0.618
FIN	-3.636***	-3.538***	1.819*	-1.119	-4.001***	-3.886***	1.724*	-0.792
OFIN	-3.596***	-4.160***	1.984**	0.974	-3.971***	-4.591***	1.914*	0.896
INTER	-2.779***	-4.154***	0.595	-0.721	-3.244***	-4.613***	0.649	-0.554
PHARMA	-2.587***	-3.097***	1.480	-1.522	-2.850***	-3.334***	1.443	-1.285
AIR	-2.589***	-4.014***	2.698***	1.394	-2.782***	-4.292***	2.567***	1.328
SCAP	-3.063***	-4.939***	3.477***	0.977	-3.493***	-5.448***	3.207***	0.928
MCAP	-3.384***	-4.275***	2.864***	1.156	-3.706***	-4.685***	2.719***	1.101
GROWTH	-3.520***	-3.829***	1.236	-0.873	-3.771***	-4.087***	1.198	-0.619
VALUE	-2.332***	-2.647***	1.216	-1.208	-2.525***	-2.847***	1.246	-0.940

Note: The tests for tail dependence and tail beta equality are defined in equations (11) and (12), respectively. Structural change is separately tested for the lower ( $l_1 = l_2$ ) and upper ( $u_1 = u_2$ ) bivariate tail, whereas lower tail-upper tail asymmetry is tested for the pre-9/11 ( $l_1 = u_1$ ) and post-9/11 ( $l_2 = u_2$ ) subsample. The equal tail beta test is conditioned upon  $p = 0.02\%$ . The test is asymptotically normal in large samples and two-sided rejections at the 10%, 5% and 2% significance level are denoted by \*, \*\* and \*\*\*, respectively.

statistical significance of the differences  $\hat{\eta}(< 9/11) - \hat{\eta}(> 9/11)$  and  $\hat{\tau}_\beta(< 9/11) - \hat{\tau}_\beta(> 9/11)$ , respectively. Thus, a significant increase in dependence and accompanying tail- $\beta$ s over time is reflected by significantly negative values of the structural change test. The asymmetry tests, on the other hand, reflect whether the differences  $\hat{\eta}(\text{lower}) - \hat{\eta}(\text{upper})$  and  $\hat{\tau}_\beta(\text{lower}) - \hat{\tau}_\beta(\text{upper})$  are statistically significant with 'lower' and 'upper' referring to the third and first return quadrant, respectively. Significantly positive values for the asymmetry test imply that there is a higher propensity toward simultaneous sectoral crashes (or 'co-crashes') than toward simultaneous booms (or 'co-booms').

A number of interesting observations can be made from the tables with test statistics. Starting with the structural change results, NYSE Composite tail- $\beta$ s significantly increased in a number of cases. Not surprisingly, sectors that have been affected by terrorism like insurance, banking and finance, airline and oil industries have become more reactive to extreme aggregate fluctuations as reflected by the NYSE Composite. Using the oil index as conditioning factor and testing for structural change, however, tail- $\beta$ s have changed even more spectacularly. All sectors seem to have become much more responsive to oil shocks in the aftermath of 9/11. As concerns the sectoral co-exceedance probabilities in Table V, they change in a statistically significant way for the old vs. new economy sector pairs, whereas nothing seems to change for the new economy index pairs. The old economy pairs seem to take an intermediate position. As concerns the tail asymmetry tests, we only found evidence of widespread asymmetry for the oil index tail- $\beta$ s. For those cases where tail asymmetry is found to be statistically significant, however, co-crashes are more likely

Table V. Structural change/asymmetry tests for sectoral co-exceedance probabilities: old vs. new economy pairs

Indices	$T_\eta[H_0: \eta_1 = \eta_2]$				$T_q[H_0: \beta_1(p) = \beta_2(p)]$			
	Structural change		Asymmetry		Structural change		Asymmetry	
	$l_1 = l_2$	$u_1 = u_2$	$l_1 = u_1$	$l_2 = u_2$	$l_1 = l_2$	$u_1 = u_2$	$l_1 = u_1$	$l_2 = u_2$
<i>Panel A: Old economy linkages</i>								
IND-TRAN	-2.090**	-3.163***	2.099**	0.118	-2.525***	-3.445***	2.00**	0.196
IND-UTIL	-2.023**	-2.011**	0.185	0.611	-2.162**	-2.197*	0.163	0.538
TRAN-UTIL	-2.746***	-2.592***	0.603	1.041	-2.962***	-2.843***	0.572	0.943
<i>Panel B: New economy linkages</i>								
PC-BIO	0.958	0.639	-0.648	-1.052	0.672	0.262	-0.242	-0.885
PC-TEL	1.108	0.778	-0.727	-1.228	1.039	0.754	-0.579	-1.01
PC-INTER	0.212	-0.070	-0.719	-1.243	0.194	-0.119	-0.514	-1.088
BIO-TEL	0.720	0.291	-0.975	-1.464	0.532	0.120	-0.614	-1.209
BIO-INTER	0.251	-0.602	-0.369	-1.744	-0.009	-0.843	0.069	-1.495
TEL-INTER	0.216	0.591	-0.799	0.029	0.162	0.475	-0.616	0.073
<i>Panel C: Old economy–New economy linkages</i>								
IND-PC	-1.422	-1.695*	-0.052	-1.078	-1.684*	-1.905*	0.021	-0.926
TRAN-PC	-3.037***	-3.284***	0.742	-0.965	-3.491**	-3.623***	0.677	-0.762
UTIL-PC	-2.367***	-2.588***	0.299	0.330	-2.627***	-2.897***	0.288	0.363
IND-BIO	-0.283	-1.965**	1.994**	-0.973	-0.721	-2.272**	1.996**	0.831
TRAN-BIO	-3.422***	-3.257***	0.894	0.474	-3.754***	-3.644***	0.886	0.470
UTIL-BIO	-1.531	-2.573***	1.202	-0.521	-1.864*	-2.899***	1.133	-0.421
IND-TEL	-1.377	-2.085**	-0.109	-1.873*	-1.623	-2.262**	-0.004	-1.700*
TRAN-TEL	-3.275***	-3.739***	0.774	-1.166	-3.718***	-4.067***	0.736	-0.906
UTIL-TEL	-2.508***	-2.516***	-0.091	0.099	-2.779***	-2.833***	-0.085	0.144
IND-INTER	-0.871	-1.723*	0.158	-1.452	-1.187	-1.922*	0.249	-1.278
TRAN-INTER	-2.004**	-3.412***	1.479	-0.530	-2.604***	-3.839***	1.380	-0.394
UTIL-INTER	-2.862***	-2.531***	-0.167	0.564	-3.130***	-2.877***	-0.127	0.579

Note: The tests for tail dependence and tail beta equality are defined in equations (11) and (12), respectively. Structural change is separately tested for the lower ( $l_1 = l_2$ ) and upper ( $u_1 = u_2$ ) bivariate tail, whereas lower tail–upper tail asymmetry is tested for the pre-9/11 ( $l_1 = u_1$ ) and post-9/11 ( $l_2 = u_2$ ) subsample. The equal tail beta test is conditioned upon  $p = 0.02\%$ . The test is asymptotically normal in large samples and two-sided rejections at the 10%, 5% and 2% significance level are denoted by \*, \*\* and \*\*\*, respectively.

than co-booms. The tables also show that asymmetries seem to vanish after 9/11. Thus, in general, the case of asymmetries seems much weaker than for structural change.<sup>25</sup>

Moreover, and parallel with the univariate results, we observe that the outcomes of the test statistics for tail dependence constancy and tail- $\beta$  constancy do not always coincide. This is because the tail- $\beta$  estimator reflects both tail dependence information ( $\eta$ ) as well as information on the scale of the auxiliary variable  $Z_{\min}(Z_{n-m,n})$ . Hence the rejection in tail- $\beta$  constancy when the tail dependence coefficient remains constant must be induced by changes in the scale of the auxiliary variable. On the other hand, if the tail- $\beta$  remains constant in the presence of significant

<sup>25</sup> For sake of comparison, we also calculated CAPM- $\beta$ s by means of truncated regressions on the tail area (see, for example, Ang and Chen, 2002). Systematic risk rankings of sectoral portfolios according to truncated CAPM- $\beta$ s and EVT-based tail- $\beta$ s are found to diverge substantially. Moreover, the outcomes of structural change and asymmetry tests for both systematic risk measures differ substantially. A possible explanation for these diverging results might be that CAPM- $\beta$ s only measure linear dependence, whereas EVT-based tail- $\beta$ s are able to capture more general return dependencies. Details of the calculations are available upon request.

tail dependence parameter changes, changes in the scale and the tail dependence parameter seem to offset each other.

#### 4. CONCLUSIONS

In this article we measure the ‘sectoral system’ risk in the US stock market by implementing multivariate extreme value estimators and tests to US sectoral index returns. We distinguish two types of measures: one capturing extremal spillovers between economic sectors (sectoral ‘co-exceedance’ probabilities) and another capturing the exposure of sectors to extreme systematic shocks (dubbed ‘tail- $\beta$ s’). We compare the relative magnitudes of these two forms of sectoral system risk across lower and upper tails and across time (i.e., are the sectoral risk measures altered in a statistically and economically significant way by 9/11?).

Our results suggest that univariate extremal tail behavior is subject to structural change. Surprisingly, structural change tests point to a decrease in left and right tail quantiles after 9/11 in a number of cases. The univariate tails also exhibit some significant asymmetries. However, and a bit counterintuitive, downside risk (as measured by the left tail quantiles) is often found to be significantly dominated by the right tail quantiles (upward potential).

Turning to the bivariate results, NYSE Composite tail- $\beta$ s in general exceed oil tail- $\beta$ s; but the statistical and economic significance of post-9/11 upward shifts in systematic risk is greatest for the latter tail- $\beta$ s. Moreover, the magnitude of extreme linkages between new economy sectors dominates pure old economy linkages or mixed old–new economy spillovers. Also, only the extreme linkages in the new economy do not exhibit an upward shift due to 9/11. Finally, empirical evidence for tail asymmetries in co-exceedance probabilities is found to be quite weak; we only found some evidence of asymmetry in the oil index tail- $\beta$ s in the pre-9/11 sample.

The observed post-9/11 rises in extreme systematic risk for certain sectors might be attributable to a ‘terrorism risk’ premium. From a regulatory point of view, the issue can be raised whether and how the current regulatory frameworks have to be adjusted to the new situation (assuming that the 9/11 effect will persist over longer time spans). From a risk management point of view, rising extreme linkages imply that the potential for sectoral risk diversification during crisis periods has decreased after 9/11.

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